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Abstract

This paper contains a review of the main paradigmatic approach to general equilibrium macroeconomic modeling, the family of New Keynesian dynamic stochastic general equilibrium models, a review of the more obscure, but recently somewhat resurgent, approach of Hyman Minsky and his Financial Instability Hypothesis, and a comparison of the two approaches. The paper then goes on to investigate the extent to which these approaches can be integrated, and to what effect.

1 Introduction

Much recent attention has been paid to assessing the paradigmatic class of general equilibrium macroeconomic models, the New Keynesian dynamic stochastic general equilibrium (DSGE) models, in the context of the financial crisis and Great Recession of 2007-2009 (for example, Gali 2018, Stiglitz 2018, Christiano et al. 2018). Criticisms raised against DSGE models include that they did not predict the financial crisis, they cannot explain the financial crisis and ensuing recession, and that they did not provide prescriptive counter-cyclical policy advice (see especially Stiglitz, 2018).

A key component to this alleged failure to adequately model the financial phenomena that drove the recession is that the most basic DSGE models do not include a financial sector or a financing mechanism; financial transactions are assumed to be efficient and frictionless so that the interaction between firms and intermediaries need not be modeled. Hence there has been much recent work to incorporate financial intermediaries and frictions in the financing process into DSGE models, such as that by Gertler and Karadi (2011) and Christiano et al. (2014), and there has been more attention paid to older DGSE models with financial frictions such as Bernanke et al. (1999) and Kiyotaki and Moore (1997).

However, even with financial frictions added, recessionary (and expansionary) deviations from the natural state of the economy in DSGE models are ultimately caused by exogenous stochastic shocks to variables such as technology or monetary policy. Thus, while DSGE models can model the dynamics of business cycle fluctuations, they cannot generate endogenous cycles, where the causes and effects of expansions are linked to the causes and effects of recessions. Furthermore, DSGE models characterize the deterministic component of the economy as fundamentally stable. Two main qualitative implications of this characterization are that: i) following any perturbation away from equilibrium, the economy will converge back to the steady state; ii) these perturbations are taken to be from completely outside the system, rather than caused by the inner-workings of the economy. Financial frictions, such as the financial accelerator of Bernanke et al. (1999), work to accentuate shocks, but do not cause the shocks.

In stark contrast, the business cycle theory of Hyman Minsky (1986) emphasizes phenomena in the financial system as the key drivers of the business cycle and economic volatility; he argues that financial instability is an inherent feature of the debt-financed, capital-intensive structure of modern economies. What occurs during expansions features in the direct causes of recessions, and vice versa, so that the business cycle ought to be viewed as a continual, endogenous, and perpetual dynamic phenomenon of modern economies—not a stochastic displacement wherein variables scramble to return to a steady state as in DSGE models.

The basic assumptions and causal mechanisms in the New Keynesian DSGE models and in Minsky's Financial Instability Hypothesis (FIH) are at their core dissimilar. Thus, the use of DSGE methodology to mathematically model a Minskian dynamic system would be incongruous. However, incorporating a minimal set of the characteristics of the FIH business cycle could aid in modeling how an endogenous build-up of instability in the financial sector of a DSGE model could trigger a financial crisis and recession. This paper will explore that potential and will proceed as follows: In section 1 I will in detail present a FIH business cycle and identify five key features of the FIH. In section 2 I will present the evolution of the DSGE paradigm, highlighting the contrast of these models with the identified FIH features. In section 3, as an exercise to investigate if aspects of the FIH can be incorporated into the DSGE framework, I will sketch an adaptation of the financial intermediation partial equilibrium portion of the Bernanke et al. (1999) DSGE to include a select set of Minskian features.

2 The Financial Instability Hypothesis

This section is divided into two parts: the first will describe the causal chain of events that constitutes a business cycle in the FIH, as represented in Minsky and Myers (1972) and Minsky (1982 and 1986). The second part will identify five key characteristics of the FIH business cycle which capture the logic of the FIH business cycle narrative.

2.1 The Minskian Business Cycle

Minsky's description of the business cycle usually begins in the proximate aftermath of a recession, or early in the recovery. At this point, the recent downturn is fresh in the minds of firms and financial intermediaries, so that the subjective probability distributions of firms over economic variables such as aggregate demand are fairly conservative and pessimistic, and financial intermediaries perceive firms and capital ventures as high-risk. Balance sheets for financial and nonfinancial firms are characterized by low leverage ratios, since the debt-deflation of the last recession induces in both types of firms an aversion to debt contracts. Firms furthermore tend to hold liquid, low-yield assets, which aided in fulfilling financial commitments during the downturn as normal cash-flows turned out to be insufficient. The important note here is that economic agents form expectations about the present and future states of the economy by extrapolating the recent past, which at this point in the narrative is distress and crisis.

As the recovery continues, aided perhaps by policy-makers, and the recession fades into memory, aggregate demand increases over that of the recessionary state that agents are using as their model to form expectations. Thus cash flows—revenue minus operating costs—prove to be sufficient for servicing the low level of debt obligations that firms at this point face. Hence it becomes clear that assessments of the level of handleable debt were too conservative coming out of the recession, firms make the profitable move toward a more leveraged financial position. Through firm competition, a business strategy that involves more leveraging will prove to the more evolutionarily fit strategy, and both financial and nonfinancial firms will assume higher degrees of debt-leveraging.

Furthermore, the valuations of capital assets begin to increase. In the FIH, capital is an asset that is valued in terms of the market value of the increase in productive capacity that the unit of capital will yield. In other words, a unit of capital will provide a stream of marginal increases in production over the life of the asset, which, by interacting with demand for the firm's output, will then provide a stream of future revenues and hence cash flows. Thus, in the FIH model, capital assets are best viewed as analogous to bonds that yield a stream of future payments, and the value of the capital asset to the firm, like the value of a bond, is equal to the discounted present-value of the future receipts. Minsky refers to the stream of cash flows thus derived from ownership of capital assets as quasi-rents. Therefore, extrapolations of the current trend of improvement in aggregate demand will increase the expected value of the future stream of quasi-rents and capital asset values, leading to an increased incentive for nonfinancial firms to obtain capital assets through debt financing.

Financial firms also extrapolate the recent low rate of default and improved profitability of capital assets to adjust to a subjective probability distribution that interprets less risk in the aggregate state of the economy. Hence a more aggressive lending strategy becomes more competitively advantageous, and financial intermediaries increase the supply of credit. This, along with the increase in demand for credit for capital expansion by firms, acts to increase the total amount of debt in the economy, as well as the total demand for capital assets.

At this stage, the increased demand for capital assets might cause the price of these assets to increase over and above their fundamental values based on quasi-rents, which could create bubbles in asset markets. This is a less emphasized phenomenon in the FIH, but Minsky discusses it (in particular, in Minsky (1986); also Rosser et al (2012)) as a channel that might introduce a significant degree of additional instability in economy. Even if bubbles do not form, the competitive pressures that push financial firms to expand credit extension also lead to incentives to innovate in creation of assets that banks can sell or use as collateral to obtain funds, in this case for the purpose of lending; Minsky refers to such assets as position-making assets. These assets are likely to be less liquid and more likely to have obscured fundamentals. Note that if these position-making assets are derived from capital assets experiencing a price bubble, or the debt contracts that fund such capital assets, then a bubble could form in the price of position-making assets.

The increase in debt-financed demand for capital assets will directly lead to an increase in the debt-financed demand for investment goods, wherein the investment goods are valued by way of analogy to the existing capital assets for which they are replacements or supplements. This increased investment leads to an even further increase in aggregate demand and output through conventional Keynesian mechanics. Through extrapolative expectations, this further still increases expected quasi-rents and thus the value of capital assets and investment goods, again inducing greater demand for debt-financed capital expansion by firms. Hence there is a positive feedback loop.

Now Minsky's taxonomy of firms by balance-sheet solvency becomes relevant. Minsky partitions firms into three classes, hedge firms, speculative firms, and Ponzi firms (this taxonomy is most clearly presented in Minsky 1986, and is also discussed in Minsky 1982). A hedge firm undertakes business ventures wherein the stream of quasi-rents from capital assets is sufficient to cover the costs of servicing and ultimately repaying the debt incurred to acquire the capital assets. A speculative firm also undertakes business ventures such that the expected value of the sum of quasi-rents is greater than or equal to the sum of payments the firm is obliged to make according to debt contracts incurred. However, over some temporary time horizon expected cash flows are positive, but are insufficient to service debt. Hence, for some period of time, speculative firms will need to roll over debt to maintain their balance sheets. Ponzi firms are firms with business ventures such that the sum of expected quasi-rents from capital assets are less than the sum of all payments necessary to service the debt incurred to obtain the capital assets (I use the notion of Ponzi firms from Minsky (1986), the definition in Minsky (1982) is slightly different).

These firms rely entirely on the prospect of capital gains to validate their existences, and at no future period do they expect to be able to service debt payments without rolling over debt. It is through the behavior of these firms that bubbles in capital asset prices are most likely to occur.

As the self-reinforcing recovery or expansion continues, competitive pressures push hedge firms to assume speculative positions, as short-term debt brings lower interest rates than long-term debt. Note that the liability column in the balance-sheets of both speculative and Ponzi firms are susceptible to increases in interest rates, since these firms rely on new debt to finance old debt; increases in the interest rates they face causes overall increases in debt and leverage that these firms have no choice but to accept.

Eventually, as both financial and nonfinancial firms' balance-sheets become more precarious with higher leverage ratios and the investment projects undertaken become more speculative, some firms, especially speculative and Ponzi firms, will be unable to recover the value of the capital assets expected at the time when debt contracts were made. Defaults will begin to occur and introduce an increase in financial intermediaries' perception of risk. This will in turn introduce risk premia shocks and increases in the interest rates that firms face, pushing speculative firms into more precarious financial positions, in which case they might no longer be able to finance debt and operating costs with revenues, and will have to run down assets to cover obligations. But, since firms have been shifting to less liquid assets throughout the expansion, the use of the of these assets to make payments could create fire-sales. To the degree that financial intermediaries are using similar assets for position-making, the fire-sales cause banks to take losses in the asset column of their balance sheets. If these firms are also highly leveraged, they might become insolvent, further increasing banks' perception of risk.

The accelerating increase in risk decreases the supply of credit. This causes a drop investment and hence aggregate demand. With extrapolative expectations, this in turn decreases expected future aggregate demand and hence the value of capital assets and investment goods. This in turn decreases the demand for debt-financed expansion, further decreasing investment and aggregate demand. Hence in the period of decline there is a negative feedback loop.

Equally important, debt contracts that even hedge firms had made before the onset of the downturn might not be validated by cashflows, since the current quasi-rents expected at the time when the debt contracts were created turn out much lower than the quasi-rents realized. If the gestation period for the production of the investment good is sufficiently long, the entire series of quasi-rents may be below that which was expected. Thus, even the most financially secure firms face the prospect of bankruptcy, and a financial crisis and deep recession may ensue.

2.2 Characteristic Features of the FIH Business Cycle

This section identifies five key characteristics of the FIH business cycle theory, selected because they represent marked deviations from conventional DSGE New Keynesian models.

First, balance sheets considerations and financial interlinkages are the ultimate causes of both the instability of expansions and the feedback loops of recessions. Instability is a direct result of endogenous buildup of risk associated with the manner in which firms finance expansionary activity, as manifested in their balance sheets. The financial interconnectedness between firms and financial intermediaries is what causes shocks to become financial crises; negative shocks to the profitability of individual firms show up in the balance sheets of not only these firms, but also their creditors, and thus have ramifications beyond those firms. Overall, insolvent balance sheets and default are key drivers of aggregate risk and financial intermediaries' perception of risk, and the probability of default is heavily influenced by leverage, the value of market-making assets, cash-flows, and how they interact.

Second, expectations are extrapolative. In the economy of the FIH, agents face Knightian uncertainty and cannot form fully specified subjective distributions over stochastic economic variables. Agents thus can only form expectations of the future by extrapolating current and recent trends, with data further in the past in the information set being deemphasized or forgotten. Subjective distributions of random variables are conditional on the state of the present and past, and hence are dynamically updated.

Third, there are significant intertemporal linkages in capital financing. Equity alone is inadequate for the financing of capital purchases and investment, hence debt contracts must be entered into. Crucially, debt obligations are made before the quasi-rents that would validate the contracts can be collected. Hence the ability to validate debt contracts are not dependent on the current state when contracts are made, but the future states of the economy, which are uncertain. Thus there will be times when the self-perceived wealth of firms, financial and nonfinancial, will be negatively corrected or nullified as firms become unable to validate debt obligations.

Fourth, at any time, there is an endogenous change in the state of the real economy and financial system, as credit and debt are built up or run down and investment projects are undertaken or abandoned. Hence in no sense is the economy ever in a steady state.

Fifth and finally, the dynamic behavior of the system is path dependent. Institutions, behaviors, subjective probabilities, and preferences such as risk aversion are dependent on at least the proximate last recession or period of expansion. If financing conditions are dependent on the recent past, then so are investment, interest rates, and the overall time-series business cycle properties of aggregate variables.

3 New Keynesian DSGE Models

This section will be split into a survey of the literature that constitutes the main thread of development of core pre-financial-crisis-DSGE models, and then a survey of the DSGE models that incorporate financial markets and frictions, both before and after the Great Recession. Both parts of this section will also contain discussion of how the assumptions and key mechanisms of these models differ from the five key FIH characteristics identified above.

3.1 The Core Pre-Crisis DSGE Literature

The goal of the New Keynesian DSGE research program is to incorporate some of the stylized facts of Keynesian theory into the Real Business Cycle (RBC) modeling framework (pioneered by Kydland and Prescott (1982), Long and Plosser (1983), and Hansen (1985)) in order to obtain numerical results that better match the empirical time series behavior of macroeconomic variables. The basic structure of DSGE models focuses on the behavior of two types of economic agents: households and firms. Households consume the output of firms and supply factors of production, and are assumed to be infinitely lived. Firms obtain factors of production from households to produce output that is used either for consumption or capital acquisition, and then return profits to households. Thus there are typically two optimization problems, one for households and one for firms. The households maximize lifetime utility, where the instantaneous utility function in each period takes consumption and leisure (as the complement of labor supplied), and sometimes cash-on-hand, as arguments. Firms maximize profits.

The maximization problems for a representative firm and a representative household generate first order conditions that give relations between aggregate economic variables with intertemporal links, since the optimization problems of firms and households are both dynamic optimization problems. These first order conditions are then log-linearized (giving the model a percent deviation interpretation) around the dynamic steady-state solution to the model. The linearized model can then be solved as a system of linear difference equation, the solution to which is then used to estimate parameters and simulate impulse responses to exogenous, stochastic shocks. These exogenous shocks, often to the productive technology of the economy or to monetary policy, are induced to perturb the system away from the steady state, and the impulse responses of the aggregate variables that comprise the models are the simulated business cycle expansions or recessions, depending on the sign of the exogenous perturbation.

The thread of DSGE models that contributed to form the canonical features of the pre-crisis DSGE models began with the models of Yun (1996), Clarida et al. (1999) and Erceg et al. (2000). These models introduced the initial separation from the RBC models by introducing monopolistic competition in the goods and labor markets. Thus, instead of labor and capital services costing their marginal productivity, and consumption goods costing their marginal costs, as would hold in the perfectly competitive RBC markets, wages and prices in these models were set as decision variables as part of households' and firms' optimization problems. In addition to monopolistic competition, another Keynesian feature that these models imposed is price and wage stickiness in the style of Calvo (1983), wherein an exogenously determined proportion of firms and households are able to reset prices or wages. Equivalently, firms and household in each period are able to reset prices and wages with an exogenously determined probability less than one. Yun (1996) introduces this mechanism in order to explain the covariance of inflation and output in response to exogenous demand-side shocks, a feature which RBC models had failed to obtain in simulations. Erceg et al. (2000) incorporate this nominal rigidity in order to be able to investigate the welfare implications of different monetary policies.

The two seminal pre-crisis DSGE models are those from Smets and Wouters (2003, 2005, 2007) and Christiano, Eichenbaum, and Evans (2001, 2005). The discussion below pertains specifically to Smets and Wouters (2003) and Christiano et al. (2005), since the models in each set are very similar, especially with respect to the topic of this paper, and these two papers have the most explicit derivation of each respective model. The main goal of these models is to capture the empirical persistence of the deviations of aggregate variables, especially inflation and output, from their steady states in response to exogenous shocks. For example, in numerical simulations in Christiano et al. (2005) output remains separated from its steady state value for approximately three years after the initial shock to monetary policy, and the authors regard achievement of this result as a success.

The features these models incorporate mostly fall on the real side of the economy. They include introducing variable capital utilization, and adjustment costs in both the capital utilization rate and the level of capital investment. On the household side of the model, these models introduce habit persistence in households' level of consumption, such that previous period consumption enters into the instantaneous utility function in addition to current consumption. In Smets and Wouters (2003), previous period consumption is represented as an aggregate variable external to the household, so that there is "keeping up with the Jones" effect. In Christiano et al (2005), the measure of previous period consumption is the household's actual consumption in the previous period.

In all of these models, financial markets are nonexistent or, in the case of Christiano et al. (2005), existent but minimal. In the models of Clarida et al. (1999) and Erceg et al. (2000), there is respectively either no capital as a factor of production, or capital is held at a constant level. Importantly, investment decisions in all of these models are made by households, which own capital and rent capital services to firms; in Smets and Wouters (2003) and Christiano et al. (2005) households decide the capital utilization rate, i.e. what proportion of capital stock each household rents out. All wealth in the economy goes to households, either through capital rents, wages, or dividends from firms' profit. Hence under the representative household approach, there is no way for a household to be a net saver or net investor, so that investment is completely financed by income and wealth. Thus there is no debt in these models, no financial interconnectedness, and there are no balance sheet considerations. In the Christiano et al. (2005) model, there is a banking sector, but its only role is to mechanically store households' wealth, lend all of the wealth to firms, which must finance production before they receive revenue from output, and pay an interest rate to households equal to the rate they charge to firms. Business cycles arising from financial crises or innovations mechanically cannot occur in these DSGE models, so that the difference between these models and the FIH model of business cycles along this characteristic alone is enough to make the two approaches incompatible.

Many of the stochastic processes in these models are assumed to take a form such that after linearization, they follow either a random walk or an AR(1) process; in either case the stochastic processes are Markov chains so that expectations of the state of the economy in the next period are based only upon the state at the present. Moreover, under the assumption that agents form rational expectations, agents' subjective probability distributions of future states are exactly equal to actual distributions. Hence the uncertainty in these models is not the Knightian uncertainty which characterizes the FIH, and agents' expectations of states far off in the future are not constantly in flux and readjusted based on the recent past. Agents and their expectations are not subject to the animal spirits brought on by euphoric good times or bearish bad times.

Also of note, in DSGE models agents do not need to form expectations far into the future as they do in the world of the FIH. In the FIH, investment is financed by debt, which needs to be serviced in a sequence future periods until the debt contract matures. By this mechanism, in the FIH the cash-flows yielded by the investment in each period in this sequence is relevant. In the baseline DSGE model, however, investment is not financed by debt, so there is no minimal cash-flow requirement that households (who in these models are the agents that undertake investment) need to meet in order to avoid the existential threat of bankruptcy. Thus the intertemporal linkages in the financing of and return on capital, which are crucial for the economy in the FIH to maintain internal coherence, are in DSGE models altogether absent.

The economy, according to the FIH, is in general always experiencing endogenous flux. In expansions (by which, in the FIH, is meant any state that is not a recession) firms are becoming more leveraged in their financing, more optimistic in their expectations of the future, and investing more aggressively; banks are becoming less risk-averse and more willing to extend credit, thus the economy is at any time undergoing real and financial changes to become both more dynamic and precarious; in recessions the inverse is true. Crucially, this flux is due to the inner workings of the economy—the decisions of economic agents. In DSGE models, recessions and expansions are symmetric deviations from a steady state induced by exogenous shocks. The implication here is that without any perturbations external to the system, the economy would be essentially be stable and static, or at least evolving according to an unvarying trend. The difference between the two approaches here can be attributed to diametrically opposed assumptions regarding the basic functioning of capitalist systems: whether they have globally stable fixed points, or whether they are subject to forces that cause internal “incoherence”, using Minsky’s (1986) phrasing.

Finally, Minsky emphasizes how institutions, especially financial institutions, undergo structural changes in response to the state of the economy in recent history. An example might be innovations in a certain type of financial product, and widespread adoption of that product for market-making purpose, which is what occurred in the market for mortgage-backed securities in the run up to the financial crisis. Other features of the economy, such as subjective probabilities and preferences, exhibit path dependence. This contrasts with DSGE models, in which the behavior of and relationships between variables are fixed for all time, according to the first-order conditions and log-linearizations. Hence expansions or recessions induced by identical shocks will be identical, provided that the system is given sufficient time to converge back to the steady-state since the last exogenous shock. The qualitative behavior of the economy is thus fixed for all time.

3.2 Financial DSGE Models

This section will highlight a few selected DSGE models that incorporate financial sectors and financial frictions, and will the discuss the extent to which these models capture the five FIH characteristics. The two canonical pre-crisis general equilibrium models that emphasize the role of financial intermediation in driving business cycles are Kiyotaki and Moore (1997) and Bernanke et al. (1999).

Kiyotaki and Moore (1997) present a highly stylized model that investigates the dynamic interaction between credit limits and asset prices in an environment where lenders cannot force borrowers to repay debts unless debts are secured by assets serving as collateral. Asset prices, which positively depend on future asset prices, affect access to credit through this collateral requirement channel. Access to credit then affects demand for capital, which feeds back into asset prices in the current period, as well as future output and asset prices, which then in turn affects future access to credit. Hence there is an important intertemporal link in this model that causes the effect of asset price shocks to persist as net worth gradually converges back to its steady-state level. The persistence in turn augments the initial response to the shock through connection between present and future values. Furthermore, shocks in this model cause dampened oscillations around the steady state, rather than the monotonic or hump-shaped convergence that characterizes conventional DSGE models. This allows the model to capture a sense of path dependence, in which a period of high capital ownership by borrowing firms and high leverage is followed by a period in which the leveraged state of firms makes investment financing difficult and capital ownership lower; the authors compare this cyclic behavior to a predator-prey model. However, in this model agents have perfect foresight, so that the fluctuations are essentially deterministic, and absent any shocks the economy will still remain in a steady state in which there are no endogenous dynamics or buildup of instability.

The Bernanke et al. (1999) model is much less stylized and is more similar to the standard DSGE model. The goal of this model is to incorporate a financial sector into an otherwise conventional DSGE model in which entrepreneurs have to borrow from banks in order to acquire capital, and face a premium in the interest rate they pay over the riskless rate in the economy. The key feature of this model is that the interest rate facing entrepreneurs is increasing in their leverage ratio, so that the balance sheet condition of the borrower is crucial in determining borrowing, capital demand, investment and output. Conversely, the net worth of entrepreneurs depends positively on output and investment, creating a feedback loop known as the financial accelerator. However, the negative relationship between leverage and access to capital is static, unlike in the FIH where preferences and expectations are adaptively based on the past allowing for endogenously emerging periods of exuberance or conservatism wherein leverage and access to capital move together.

Christiano et al. (2014) develop the concept of the Bernanke et al. (1999) model with the goal of incorporating a risk shock, or a shock to the standard deviation of the firm specific rate of return on capital, a random variable in both Bernanke et al (1999) and in Christiano et al. (2014). Here too balance sheet considerations are central; the interest rate that entrepreneurs face is positively related to the leverage ratio and the aggregate risk represented by the standard deviation of firm-idiosyncratic returns. A key departure from the FIH is that financial intermediaries are assumed to be perfectly diversified across entrepreneurs, so that by the law of large numbers intermediary returns are equal to expected returns, implying that intermediaries cannot become insolvent and there can be no economy-wide loss of wealth. As a final note, in both Bernanke et al. (1999) and Christiano et al. (2014) entrepreneurial projects are one-period projects; firms obtain funds from banks, purchase capital, use the capital to produce output in Bernanke et al. (1999) or to transform raw capital into effective capital in Christiano et al. (2014), then resell nondepreciated capital and begin the entire process anew next period. Hence the intertemporal links in these models are weak—the expected state of the economy years into the future does not have a great effect on current financial market conditions, and financial commitments made periods in the past play no role.

All of the above models focus on frictions arising in the market for credit between financial intermediaries and nonfinancial firms. Given that the financial crisis was caused by frictions between financial intermediaries, many post-crisis financial DSGE models have incorporated frictions in the market where financial intermediaries obtain deposits from households. A few papers in this vein include Gertler and Karadi (2011), Gertler and Kiyotaki (2015) and Gertler et al. (2016).

The goal of Gertler and Karadi (2011) is to model the effect of balance-sheet deterioration of financial intermediaries on credit extension, through both credit constraints as in Kiyotaki and Moore (1997) and through a risk premium on interest rates. In this model, banks in any period may declare bankruptcy and abscond with a fraction of deposits, so that in order for households to deposit funds in banks, lending projects must satisfy an incentive constraint in which the expected value of the bank over its lifetime is greater than what the bank could receive by stealing the deposits in the current period. Hence intertemporal links are crucial in this model, since the value of the lending projects to the bank is dependent upon the return on lending and the overall state of the economy in every future period (weighted by the probability that the bank will still exist in each period). In other words, the leverage ratio required of banks by depositors to account for moral hazard is a positive function of both the growth rate of asset prices, the spread on returns from lending to firms over the riskless return rate, and the growth rate of net worth over the entire life of the bank. A higher leverage ratio in turn leads to higher returns for banks and a higher growth rate of net worth. This link between agents' preferences toward acceptable leverage and beliefs about the future state of the economy, and also between the degree of leverage and dynamism of the economy, are central to the FIH. However, as is typical in DSGE models, expectations are rational (hence not extrapolative) and the dynamics are caused by exogenous perturbations away from the steady state, so that the business cycle is still ultimately caused by an external shock and is not attributed to the endogenous inner-workings of the economy. The result is a financial accelerator similar to that in Bernanke et al. (1999), but with greater intertemporal linkages in financial contracts.

Gertler and Kiyotaki (2015) present a partial equilibrium model of the financial sector, which is embedded into the general equilibrium model of Gertler et al. (2016). The financial sector in these models is similar to that in Gertler and Karadi (2011), with the addition that under certain balance sheet conditions a bank run may occur, which completely decimates the financial sector. In this model a bank run is possible if a fire sale on capital assets would be able to drive prices low enough to make financial intermediaries insolvent. Hence this model places even greater emphasis on balance sheet conditions and the leverage ratio than any of the models discussed above, since the ability of intermediaries to take a hit in the asset column and still remain solvent depends upon how much capital they have to absorb the hit. Therefore, in addition to impacting the economy through the financial accelerator, Gertler and Kiyotaki (2015) and Gertler et al. (2016) are able to model a Minsky moment, wherein firms become unable to honor debt contracts made in the past and the financial markets of the economy become defunct, ushering in a deep recession.

4 A Partial Equilibrium Minsky Model

This section contains a sketch of a Minskian adaptation of the financial intermediation partial equilibrium portion of the model in Bernanke et al. (1999) (henceforce BGG). As shown above, the FIH and the theory guiding DSGE models are at their roots incompatible. So the alterations here are smaller changes that do not capture the essence of the FIH, but may incorporate a Minskian flavor. The main addition is to give the borrowers in this market, entrepreneurs using the terminology in BGG, the choice to operate as hedge firms or Ponzi firms. The main distinction between these two modes of operation pertain to preferences over risk and the average return on investment, with Ponzi firms having a greater preference toward risk and a lower average return than hedge firms. In short, Ponzi firms have less profitable production operations, and are mainly enticed by the prospect of capital gains.

These adjustments to the BGG model are designed to address how firms could be induced by shocks to capital asset prices to assume a Ponzi mode of operation and take on more debt to purchase more capital than more conservative preferences would warrant, and how this increased demand for capital could in turn have a positive feedback effect on asset prices. Recall that the financial accelerator behaves as follows: a positive shock to the economy increases the return on investment and hence the net worth of entrepreneurs, which lowers the interest rate premium over the riskless rate that lenders charge borrowers, which then increases demand for capital and investment, which finally causes an increase in asset prices, again increasing the next period rate of return for entrepreneurs. The question here is whether allowing preferences toward risk to vary positively with the business cycle further augments this accelerator. Analysis of the model presented here suggests that this is possible, although numerical simulations would be necessary to truly evaluate the effects.

This subsection will first present the model and derive the main relationships between variables, then will analyze these relationships to evaluate the possibility that the dynamics added in this paper will give rise to an additional accelerator effect.

4.1 Model Exposition

The model consists of entrepreneurs, banks, and perfectly competitive capital production firms. Entrepreneurs are finitely lived agents that purchase capital using their own equity (net worth) and funding borrowed from banks. Entrepreneurs survive into next period with probability δ, so that their expected life span is $\frac{1}{1-\delta}$. After using capital to produce, they then sell capital to the capital production firms, which use the capital to invest and create new capital. Each new entrepreneur is by default a hedge entrepreneur, and in each period has the choice to become a Ponzi entrepreneur for the next period. What the firm type means will be explained below.

More explicitly, in each period t four events occur sequentially. First, firms produce with capital acquired in period $t-1$. The demand side of the economy will be taken as exogenous, and it will be assumed that all markets for firms' goods clear and firms are able to sell all output, so that at this point in t firms realize returns from production.

Second, capital is sold to the capital production firms, which are perfectly competitive and have a constant returns to scale technology. At this point in t capital gains for entrepreneurs are realized, and thus so fully are returns. Also, since banks are risk-averse and an entrepreneur is risk neutral or risk loving, depending respectively on whether the entrepreneur is hedge or Ponzi, entrepreneurs bear all the risk and interest rates are contingent on actualized aggregate returns. Hence the interest rates for the last period are here determined.

Third, entrepreneurs decide whether to be Ponzi or hedge firms, and net worth is updated. Fourth and last, the contracts for period $t+1$ are determined, as is capital demanded by each firm for $t+1$. Capital production (CP) firms invest to meet the capital demanded. It should also be noted here that each firm type has a different contracting and investment problem, since capital demanded will be contingent on the objective function for the entrepreneur, which varies with type. The events in the sequence will now be presented in full detail.

4.1.1 Rates of Return from Production

The rate of return for Ponzi entrepreneur j in period $t+1$ is

$$R^{pj}_{t+1} = \frac{\omega^{p,j} Y}{K^j_{t+1}} + \gamma_{t+1} \frac{\overline{Q}_{t+1}}{Q_t} \equiv \omega^{p,j} y^j_{t+1} + \gamma_{t+1} q_{t+1},$$

where ω^p is a random variable that represents the idiosyncratic risk that Ponzi firm j faces, $\{\gamma_t\}$ is an iid, mean one stochastic process, Y is the exogenous level of demand,

K^j_{t+1} is the capital level of firm j, $\overline{Q}_{t+1}$ is the (market) price at which firm j sells capital to the CP firms in period $t+1$, and Q_t is the (market) price at which the firm purchases capital from CP firms in period t. The idiosyncratic risk term $\omega^{p,j}$ captures the distribution over the rate of return for firm j's business venture *independent* of the aggregate state of the economy. Let, for all Ponzi firms j,

$$E_t[\omega^{p,j}] \equiv \rho < 1.$$

Then the aggregate or expected rate of return (expectations taken over the idiosyncratic risk only) for Ponzi firms is

$$R^p_{t+1} = \rho y^j_{t+1} + \gamma_{t+1} q_{t+1}. \tag{1}$$

Similarly, the rate of return for hedge entrepreneur j is

$$R^{h,j}_{t+1} = \omega^{h,j} y^j_{t+1} + \gamma_{t+1} q_{t+1}.$$

Assuming

$$E_t[\omega^{h,j}] = 1,$$

the aggregate or expected rate of return across hedge firms is

$$R^h_{t+1} = y^j_{t+1} + \gamma_{t+1} q_{t+1}. \tag{2}$$

Define the return premia over the riskless rate as

$$s_{t+1}^{p,j} = \frac{R_{t+1}^{p,j}}{R_{t+1}}; \quad s_{t+1}^{h,j} = \frac{R_{t+1}^{h,j}}{R_{t+1}}, \tag{3}$$

where R_{t+1} is the riskless rate of return. Lastly, the pdf and cdf for $\omega^{i,j}$ will respectively be denoted f^i and F^i, for $i = p, h$

4.1.2 Capital Production Market

The specification of the capital production market follows BGG very closely. CP firms purchase output at a price of 1 to be used as raw material for new capital production; this raw material input is denoted I_t. CP firms produce according to the technology (assuming, for convenience no depreciation)

$$K_{t+1} = \varphi\left(\frac{I_t}{K_t}\right)K_t + K_t; \quad \varphi'(\cdot) > 0, \varphi''(\cdot) < 0, \varphi(0) = 0.$$

The amount of raw material needed is implicitly determined by aggregate capital demanded by entrepreneurs, K_{t+1}.

The profit function for any CP firm is

$$\pi = Q_t\varphi\left(\frac{I_t}{K_t}\right)K_t - I_t - \overline{Q}_t K_t + Q_t K_t,$$

and the first order condition with respect to I_t is

$$Q_t = \left[\varphi'\left(\frac{I_t}{K_t}\right)\right]^{-1}. \tag{4}$$

Since $\varphi : \mathbb{R}^+ \cup \{0\} \to \mathbb{R}^+ \cup \{0\}$ is monotonic increasing, and assuming that it is also surjective, φ has an inverse $\hat{\theta} : \mathbb{R}^+ \cup \{0\} \to \mathbb{R}^+ \cup \{0\}$ such that $\hat{\theta}'(\cdot) > 0$, $\hat{\theta}''(\cdot) > 0$, and $\hat{\theta}(0) = 0$. Then

$$\frac{I_t}{K_t} = \hat{\theta}\left(\frac{K_{t+1} - K_t}{K_t}\right),$$

and

$$Q_t = \left[\varphi'\left(\hat{\theta}\left(\frac{K_{t+1} - K_t}{K_t}\right)\right)\right]^{-1} \equiv \theta\left(\hat{k}_{t+1}\right), \tag{5}$$

where $\hat{k}_{t+1} \equiv \frac{K_{t+1} - K_t}{K_t}$. Note that

$$\theta'(\hat{k}_{t+1}) = -\hat{\theta}'(\hat{k}_{t+1})\varphi''\left(\hat{\theta}(\hat{k}_{t+1})\right)\left[\varphi'\left(\hat{\theta}(\hat{k}_{t+1})\right)\right]^{-2} > 0,$$

since $\varphi''(\cdot) < 0$. Thus Q_{t+1} is increasing in $\hat{k}_{t+1}$.

Since firms have constant returns to scale, they must have no profit in equilibrium. This condition is

$$0 = Q_t \varphi\left(\frac{I_t}{K_t}\right) - \frac{I_t}{K_t} - (\overline{Q}_t - Q_t).$$

With the parameterization in BGG, this no profit condition implies that $\overline{Q}_t = Q_t$ in the steady state. This equality will be maintained for the rest of the model. Capital gains is thus given by

$$q_{t+1} \equiv \frac{Q_{t+1}}{Q_t} = \frac{\theta(\hat{k}_{t+2})}{\theta(\hat{k}_{t+1})}. \tag{6}$$

Hence for positive capital gains to be realized, the rate of capital growth must be increasing. With the realization of capital gains, along with returns from production, given above, entrepreneurial net worth going into the next period can be updated. However, the exposition is will be clearer if the law of motion for net worth is presented at the end, since it involves terms from the contracting problem.

4.1.3 Market for Loanable Funds

This subsection contains the mechanics of the loanable funds market; the solution to the optimal contracting problem will be derived below.

Entrepreneur j of either type $(i = h, p)$ has net worth at the end of period t of N^j_{t+1}. Thus, the amount borrowed is

$$B^j_{t+1} = Q_t K^j_{t+1} - N^j_{t+1}.$$

As mentioned above, entrepreneurs are either risk neutral or risk loving, so that they absorb all aggregate risk. Hence the interest rate is state-contingent on the actualized value of R^i_{t+1}. If the contractual interest rate on the loan to j is Z^j_{t+1}, the threshold level of ω^j_{t+1}, $\overline{\omega}^j_{t+1}$, that separates default from solvency can be defined

$$\overline{\omega}^j_{t+1} K^j_{t+1} R^i_{t+1} Q_t = Z_{t+1} B^j_{t+1},$$

so that the contract on the borrowed funds may be put in terms of $\overline{\omega}^j_{t+1}$ rather than Z_{t+1}.

Banks obtain funds from depositors, who are guaranteed a rate of return equal to the riskless rate, R_{t+1}. Banks will also be assumed to be perfectly competitive, so that in equilibrium they make no profit. Moreover, there is an informational asymmetry in this market in that banks cannot observe the return of entrepreneurs in the case of default without paying a monitoring cost of $\mu \in (0,1)$ times the total return to firms. Hence the banks' constraint on lending is

$$\left[(1 - F^i(\overline{\omega}^j_{t+1}))\right] Z^j_{t+1} B^j_{t+1} + (1-\mu) \int_0^{\overline{\omega}^j_{t+1}} \omega R^i_{t+1} Q_t K^j_{t+1} f^i(\omega) d\omega = R_{t+1} B^j_{t+1}$$

Combining the above three equations, the lender's constraint can ultimately be put in terms of $\overline{\omega}^j_{t+1}$ rather than Z^j_{t+1}

$$\left[(1 - F^i(\overline{\omega}^j_{t+1}))\overline{\omega}^j_{t+1} + (1-\mu) \int_0^{\overline{\omega}^j_{t+1}} \omega f^i(\omega) d\omega\right] R^i_{t+1} Q_t K^j_{t+1} = R_{t+1}(Q_t K^j_{t+1} - N_{t+1}). \tag{7}$$

The expected return to entrepreneur j is

$$E\left[\int_{\overline{\omega}^j_{t+1}}^{\infty} \omega R^i_{t+1} Q_t K^j_{t+1} f^i(\omega) d\omega - (1 - F^i(\overline{\omega}^j_{t+1}))\overline{\omega}^j_{t+1} u R^i_{t+1} Q_t K^j_{t+1}\right].$$

This can be simplified with (7):

$$E\left[\int_{\overline{\omega}^j_{t+1}}^{\infty}\omega R^i_{t+1}Q_tK^j_{t+1}f^i(\omega)d\omega-(1-F^i(\overline{\omega}^j_{t+1}))\overline{\omega}^j_{t+1}R^i_{t+1}Q_tK^j_{t+1}\right] \quad (8)$$

$$=E\left[\left(1-\mu\int_0^{\overline{\omega}^j_{t+1}}\omega f^i(\omega)d\omega\right)R^i_{t+1}Q_tK^j_{t+1}\right]-R_{t+1}(Q_tK^j_{t+1}-N^j_{t+1}). \quad (9)$$

To conclude this subsection, terms will be defined to simplify equations (7) and (8) and (9). Define $\Gamma_i(\overline{\omega}^j_{t+1})$ to be the gross expected share of the entrepreneur's profit that the lender will obtain as revenue from firm j of type i,

$$\Gamma_i(\overline{\omega}^j_{t+1})=\int_0^{\overline{\omega}^j_{t+1}}\omega f^i(\omega)d\omega+\overline{\omega}^j_{t+1}\int_{\overline{\omega}^j_{t+1}}^{\infty}f^i(\omega)d\omega.$$

Note that

$$\begin{aligned}\Gamma'_i(\overline{\omega}^j_{t+1})&=\overline{\omega}^j_{t+1}f^i(\overline{\omega}^j_{t+1})+\int_{\overline{\omega}^j_{t+1}}^{\infty}f^i(\omega)d\omega-\overline{\omega}^j_{t+1}f^i(\overline{\omega}^j_{t+1})\\&=1-F^i(\overline{\omega}^j_{t+1}),\end{aligned}$$

and

$$\Gamma''_i(\overline{\omega}^j_{t+1})=-f^i(\overline{\omega}^j_{t+1}).$$

Also define

$$G_i(\overline{\omega}^j_{t+1})=\int_0^{\overline{\omega}^j_{t+1}}\omega f^i(\omega)d\omega,$$

and note that

$$\mu G_i'(\overline{\omega}_{t+1}^j) = \mu \overline{\omega}_{t+1}^j f^i(\overline{\omega}_{t+1}^j).$$

The expected net share of profits that lenders will receive is then $\Gamma(\overline{\omega}_{t+1}^j) - \mu G(\overline{\omega}_{t+1}^j)$, and the expected share received by borrowers is $1 - \Gamma(\overline{\omega}_{t+1}^j)$.

A couple of results shown in BGG that will be key in the analysis below are: i) $\Gamma_i(\overline{\omega}_{t+1}^j) - \mu G_i(\overline{\omega}_{t+1}^j)$ obtains a maximum value; ii) for the parameterizations in BGG the value of $\overline{\omega}_{t+1}^j$ that solves the contracting problem is always less than the value of $\overline{\omega}$ that maximizes $\Gamma_i(\overline{\omega}_{t+1}^j) - \mu G_i(\overline{\omega}_{t+1}^j)$, so banks do not ration credit; and iii)

$$\Gamma_i'(\overline{\omega}_{t+1}^j)\mu G_i''(\overline{\omega}_{t+1}^j) - \Gamma_i''(\overline{\omega}_{t+1}^j) G_i'(\overline{\omega}_{t+1}^j) > 0 \tag{10}$$

4.1.4 Firm-Mode Decision Rule

Hedge firm j's preference function is given by

$$u^{h,j} = E_t \left[(1 - \Gamma_h(\overline{\omega}_{t+1}^j)) s_{t+1}^h k_{t+1}^j \right],$$

and Ponzi firm j's preference function is given by

$$u^{p,j} = E_t \left[(1 - \Gamma_p(\overline{\omega}_{t+1}^j)) s_{t+1}^p k_{t+1}^j \right] + \frac{\alpha^j}{2} Var_t \left[(1 - \Gamma_p(\overline{\omega}_{t+1}^j)) s_{t+1}^p k_{t+1}^j \right],$$

where $k_{t+1}^j \equiv \frac{Q_t K_{t+1}^j}{N_{t+1}^j}$, and α^j is distributed cross-sectionally across all firms according to the distribution f^α. A firm will be enticed to switch from hedge to Ponzi if the latent Ponzi preference is greater than the hedge preference; that is, if

$$E_t\left[(1-\Gamma_p(\overline{\omega}^j_{t+1}))s^p_{t+1}k^j_{t+1}\right]+\frac{\alpha^j}{2}Var_t[(1-\Gamma_p(\overline{\omega}^j_{t+1}))s^p_{t+1}k^j_{t+1}]>E_t\left[(1-\Gamma_h(\overline{\omega}^j_{t+1}))s^h_{t+1}k^j_{t+1}\right],$$

or

$$\alpha^j>2\frac{E_t[(1-\Gamma_h(\overline{\omega}^j_{t+1}))s^h_{t+1}-(1-\Gamma_p(\overline{\omega}^j_{t+1}))s^p_{t+1}]}{k^j_{t+1}Var_t[(1-\Gamma_p(\overline{\omega}^j_{t+1}))s^p_{t+1}]}$$

At this point, there is problem in that the right-hand side above depends on $t+1$ choice variables such as capital demanded, and capital demanded at $t+1$ will depend on the entrepreneur type, which is what is currently being decided. Hence assume that firms make the decision using counterfactuals based on present period variables. This is a stylized behavior in keeping with the Minskian spirit of this model: that people form expectations and make decisions based on what occurred in the recent past by extrapolating the past into the present.

Define the variable

$$\alpha^*_{t+1}\equiv\frac{2N^j_tE_{t-1}[(1-\Gamma_h(\overline{\omega}^j_t))s^h_t-(1-\Gamma_p(\overline{\omega}^j_t))s^p_t]}{K^j_tQ_{t-1}Var_t[(1-\Gamma_p(\overline{\omega}^j_t))s^p_t]} \tag{11}$$

to be the lower threshold above which firm j would be induced to switch to become a Ponzi firm. Note that α^*_{t+1} is decreasing in the variance of rate of returns received by the entrepreneur and in leveraged capital, and is increasing in the difference of returns from production between hedge and Ponzi firms. Explicitly, the proportion of firms that are Ponzi is given by

$$\phi_{t+1} = \int_{\alpha^*_{t+1}}^{\infty} f^{\alpha}(\alpha) d\alpha. \tag{12}$$

Note that ϕ_{t+1} is decreasing in α^*_{t+1}, and is thus increasing in the variance of returns and in the level and price of capital, and is decreasing in net worth and the returns to production gap between hedge and Ponzi firms.

4.1.5 Contracting Problem

The contracting problem will be solved for only the Ponzi firm, because the problem for the hedge firm is a simpler particular case of that for the Ponzi firm (where $a^j = 0$), and is otherwise already derived in BGG. For this reason and for notational convenience, the subscripts on Γ and G will be dropped. The problem consists of maximizing the expected preference function of the entrepreneur (9) such that the equilibrium constraint of the lender (7) is satisfied:

$$\max_{\overline{\omega}^j_{t+1}, K^j_{t+1}} E_t \left[(1 - \Gamma(\overline{\omega}^j_{t+1})) s^p_{t+1} k^j_{t+1} + \frac{\alpha^j}{2} Var_t \left[(1 - \Gamma(\overline{\omega}^j_{t+1})) s^p_{t+1} k^j_{t+1} \right] \right] \tag{13}$$

$$\text{s.t} \left[(1 - F(\overline{\omega}^j_{t+1})) \overline{\omega}^j_{t+1} + (1 - \mu) \int_0^{\overline{\omega}^j_{t+1}} \omega f(\omega) d\omega \right] u R^k_{t+1} Q_t K^j_{t+1} = R_{t+1} (Q_t K^j_{t+1} - N_{t+1}). \tag{14}$$

This problem can be expressed more parsimoniously using the notation developed in the market for loanable funds subsection. In Lagrangian form, this is

$$\max_{\overline{\omega}^j_{t+1},k^j_{t+1}} E_t\left[(1-\Gamma(\overline{\omega}^j_{t+1}))s^p_{t+1}k^j_{t+1}+\frac{\alpha^j}{2}Var_t\left[(1-\Gamma(\overline{\omega}^j_{t+1}))s^p_{t+1}k^j_{t+1}\right]\right]$$
$$+\lambda\left[(1-\Gamma(\overline{\omega}^j_{t+1}))k_{t+1}s^p_{t+1}-(k_{t+1}-1)\right].$$

The first order conditions are

$$\overline{\omega}^j_{t+1}:0=\Gamma'(\overline{\omega}^j_{t+1})+\alpha^j\Gamma'(\overline{\omega}^j_{t+1})(1-\Gamma(\overline{\omega}^j_{t+1}))k^j_{t+1}\frac{Var_t\left[s^p_{t+1}\right]}{\left[s^p_{t+1}\right]}-\lambda\left(\Gamma'(\overline{\omega}^j_{t+1})-\mu G'(\overline{\omega}^j_{t+1})\right) \tag{15}$$

$$k^j_{t+1}:0=E_t\left[\Lambda s^p_{t+1}-\lambda+\alpha^j k^j_{t+1}Var_t\left[(1-\Gamma(\overline{\omega}^j_{t+1}))s^p_{t+1}\right]\right] \tag{16}$$

$$\lambda:0=\left(\Gamma(\overline{\omega}^j_{t+1})-\mu G(\overline{\omega}^j_{t+1})\right)k^j_{t+1}s^p_{t+1}-(k_{t+1}-1). \tag{17}$$

$$\Lambda\equiv(1-\Gamma(\overline{\omega}^j_{t+1}))+\lambda\left(\Gamma(\overline{\omega}^j_{t+1})-\mu G(\overline{\omega}^j_{t+1})\right). \tag{18}$$

The first order conditions for hedge firms are obtained by setting $\alpha^j=0$ and replacing each superscript p with an h.

Note that the budget constraint is not in expectations. This is because since the entrepreneur assumes all risk, $\overline{\omega}^j_{t+1}$ adjusts in response to aggregate changes such that the bank's budget constraint holds. Hence $\overline{\omega}^j_{t+1}$ is really a function of (k_{t+1},s^p_{t+1}). For this same reason, the first order condition for $\overline{\omega}^j_{t+1}$ is not in expectations, and λ is the Lagrange multiplier given the aggregate return for Ponzi firms. This is analogous to the specification in BGG. All of the components of the model have now been presented.

4.2 Model Analysis

With the model presented above, the potential mechanism by which the BGG financial accelerator could be augmented will be explicitly demonstrated. The only stochastic process in this model is $\{\gamma_t\}$, hence the only shock in this model is one to asset prices. By the mechanisms in BGG, a positive shock to asset prices would raise returns above the expected level, which would then increase net worth and decrease the capital-equity ratio k^j_{t+1}. It will be shown below that the return ratio over the riskless rate is positively related to the capital-equity ratio. The only component of the return ratio that is at this point variable is the capital level, which has a negative effect on the return ratio since capital in the production function has decreasing marginal product. Thus the increase in net worth will cause an increase in capital, since the returns ratio is positively related to the capital-equity ratio. However, capital will increase by less than would be necessary to bring the return ratio back to its initial level, since the capital-equity ratio is also increasing in capital; the return ratio would then be lower than its initial level. Therefore the capital-equity ratio will also be lower than its initial level. The increase in next period capital further induces an increase in asset prices according to (5), which acts as a multiplier on the original shock to asset prices.

The above process is fully captured in BGG. The additional dynamics in this model introduced by the inclusion of Ponzi firms depends on how the relationship between the return-ratio and the capital-equity ratio differs between hedge and Ponzi firms, and how the proportion of Ponzi firms ϕ_t is affected by the positive shock.

These questions will be addressed in order.

4.2.1 Contracting Problem

This subsection will investigate how the relationship between the capital-equity ratio k^j_{t+1} and the rate of return over the riskless rate differs for hedge and Ponzi firms. Consider the contracting problem for the Ponzi firm. Total differentiation of equation (17) gives

$$0 = E_t\left[\Lambda + s^p_{t+1}\frac{\partial\Lambda}{\partial\overline{\omega}^j_{t+1}}\frac{\partial\overline{\omega}^j_{t+1}}{\partial s^p_{t+1}} - \frac{\partial\lambda}{\partial\overline{\omega}^j_{t+1}}\frac{\partial\overline{\omega}^j_{t+1}}{\partial s^p_{t+1}} + \alpha^j k^j_{t+1}\frac{dVar_t[(1-\Gamma(\overline{\omega}^j_{t+1}))s^p_{t+1}]}{ds^p_{t+1}} - \frac{\partial\lambda}{\partial s^p_{t+1}}\right]ds^p_{t+1}$$
$$+ E_t\left[s^p_{t+1}\frac{\partial\Lambda}{\partial\overline{\omega}^j_{t+1}}\frac{\partial\overline{\omega}^j_{t+1}}{\partial k^j_{t+1}} - \frac{\partial\lambda}{\partial\overline{\omega}^j_{t+1}}\frac{\partial\overline{\omega}^j_{t+1}}{\partial k^j_{t+1}} + \alpha^j\frac{dk^j_{t+1}Var_t[(1-\Gamma(\overline{\omega}^j_{t+1}))s^p_{t+1}]}{dk^j_{t+1}} - \frac{\partial\lambda}{\partial k^j_{t+1}}\right]dk^j_{t+1},$$

or

$$\frac{ds^p_{t+1}}{dk^j_{t+1}} = \frac{E_t\left[\left(\frac{\partial\lambda}{\partial\overline{\omega}^j_{t+1}} - s^p_{t+1}\frac{\partial\Lambda}{\partial\overline{\omega}^j_{t+1}}\right)\frac{\partial\overline{\omega}^j_{t+1}}{\partial k^j_{t+1}} + \frac{\partial\lambda}{\partial k^j_{t+1}} - \alpha^j\frac{dk^j_{t+1}Var_t[(1-\Gamma(\overline{\omega}^j_{t+1}))s^p_{t+1}]}{dk^j_{t+1}}\right]}{E_t\left[\Lambda + \left(s^p_{t+1}\frac{\partial\Lambda}{\partial\overline{\omega}^j_{t+1}} - \frac{\partial\lambda}{\partial\overline{\omega}^j_{t+1}}\right)\frac{\partial\overline{\omega}^j_{t+1}}{\partial s^p_{t+1}} - \frac{\partial\lambda}{\partial s^p_{t+1}} + \alpha^j k^j_{t+1}\frac{dVar_t[(1-\Gamma(\overline{\omega}^j_{t+1}))s^p_{t+1}]}{ds^p_{t+1}}\right]}. \tag{19}$$

The analogous equation for the hedge case is

$$\frac{ds^h_{t+1}}{dk^j_{t+1}} = \frac{E_t\left[\left(\frac{\partial\lambda}{\partial\overline{\omega}^j_{t+1}} - s^h_{t+1}\frac{\partial\Lambda}{\partial\overline{\omega}^j_{t+1}}\right)\frac{\partial\overline{\omega}^j_{t+1}}{\partial k^j_{t+1}}\right]}{E_t\left[\Lambda + \left(s^h_{t+1}\frac{\partial\Lambda}{\partial\overline{\omega}^j_{t+1}} - \frac{\partial\lambda}{\overline{\omega}^j_{t+1}}\right)\frac{\partial\overline{\omega}^j_{t+1}}{\partial s^h_{t+1}}\right]}, \tag{20}$$

which comes from setting α^j to zero and from the fact that, in the hedge case, λ is a function of $\overline{\omega}^j_{t+1}$ only. It is shown in BGG that $\frac{s^h_{t+1}}{dk^j_{t+1}} > 0$. However, the analysis below will show that, in this model, $\frac{ds^p_{t+1}}{k^j_{t+1}}$ has a sign that is ambiguous. If the return ratio decreases in the capital-equity ratio for Ponzi firms, then in response to an increase in net worth Ponzi firms will have to *decrease* capital to raise the return ratio. This would work to decrease the financial accelerator. In general, if $\frac{ds^p_{t+1}}{dk^j_{t+1}} > \frac{ds^h_{t+1}}{dk^j_{t+1}}$, then Ponzi firms would increase capital by more than hedge firms in response to a shock to net worth. If this is the case, then the presence of Ponzi firms augments the financial accelerator. If not, then the effect is a dampening. Each of the differential terms in (20) will now be investigated further.

First, by (16) λ can be expressed as a function of $(k^j_{t+1}, s^p_{t+1}, \overline{\omega}^j_{t+1})$,

$$\lambda = \frac{\Gamma'(\overline{\omega}^j_{t+1}) + \alpha^j k^j_{t+1}\Gamma(\overline{\omega}^j_{t+1})\left(1-\Gamma(\overline{\omega}^j_{t+1})\right)\left(Var_t[s^p_{t+1}]/s^p_{t+1})\right)}{\Gamma'(\overline{\omega}^j_{t+1}) - \mu G'(\overline{\omega}^j_{t+1})}. \tag{21}$$

Then differentiating with respect to s^p_{t+1} and k^j_{t+1} gives

$$\frac{\partial \lambda}{\partial s^p_{t+1}} = -\frac{\alpha^j k^j_{t+1}\Gamma'(\overline{\omega}^j_{t+1})\left(1-\Gamma(\overline{\omega}^j_{t+1})\right)Var_t[s^p_{t+1}]}{\left(\Gamma'(\overline{\omega}^j_{t+1}) - \mu G'(\overline{\omega}^j_{t+1})\right)s^{p2}_{t+1}} < 0 \tag{22}$$

$$\frac{\partial \lambda}{\partial k^j_{t+1}} = \frac{\alpha^j \Gamma'(\overline{\omega}^j_{t+1})\left(1-\Gamma(\overline{\omega}^j_{t+1})\right)Var_t[s^p_{t+1}]}{\left(\Gamma'(\overline{\omega}^j_{t+1}) - \mu G'(\overline{\omega}^j_{t+1})\right)} > 0. \tag{23}$$

(23) enters negatively into the denominator of (20) and (24) enters positively in the numerator of (20), so the ultimate effects of these terms on $\frac{s^p_{t+1}}{k^j_{t+1}}$ offset to a degree.

Next, differentiating (18) with respect to s^p_{t+1} and rearranging gives

$$\frac{\partial \overline{\omega}^j_{t+1}}{\partial s^p_{t+1}} = \frac{-(\Gamma(\overline{\omega}^j_{t+1}) - \mu G(\overline{\omega}^j_{t+1}))}{(\Gamma'(\overline{\omega}^j_{t+1}) - \mu G'(\overline{\omega}^j_{t+1}))s^p_{t+1}} < 0, \tag{24}$$

and doing the same with respect to k^j_{t+1},

$$\frac{\partial \overline{\omega}^j_{t+1}}{\partial k^j_{t+1}} = \frac{1-(\Gamma(\overline{\omega}^j_{t+1})-\mu G(\overline{\omega}^j_{t+1}))s^p_{t+1}}{(\Gamma'(\overline{\omega}^j_{t+1})-\mu G'(\overline{\omega}^j_{t+1}))k^j_{t+1}s^p_{t+1}} = \frac{1/k^j_{t+1}}{(\Gamma'(\overline{\omega}^j_{t+1})-\mu G'(\overline{\omega}^j_{t+1}))k^j_{t+1}s^p_{t+1}} > 0, \tag{25}$$

the second equality from (18).

Computing

$$\begin{aligned}\frac{\partial \Lambda}{\partial \overline{\omega}^j_{t+1}} &= -\Gamma'(\overline{\omega}^j_{t+1}) + \frac{\partial \lambda}{\partial \overline{\omega}^j_{t+1}}\left(\Gamma(\overline{\omega}^j_{t+1} - \mu G(\overline{\omega}^j_{t+1}))\right) + \lambda\left(\Gamma'(\overline{\omega}^j_{t+1}) - \mu G'(\overline{\omega}^j_{t+1})\right)\\ &= \alpha^j k^j_{t+1}\Gamma'(\overline{\omega}^j_{t+1})\left(1-\Gamma(\overline{\omega}^j_{t+1})\right)\frac{Var_t[s^p_{t+1}]}{s^p_{t+1}} + \frac{\partial \lambda}{\partial \overline{\omega}^j_{t+1}}\left(\Gamma(\overline{\omega}^j_{t+1}) - \mu G(\overline{\omega}^j_{t+1})\right),\end{aligned}$$

the second equality by (21). Hence, using (18) again for the second equality,

$$\begin{aligned}&\frac{\partial \lambda}{\partial \overline{\omega}^j_{t+1}} - s^p_{t+1}\frac{\partial \Lambda}{\partial \overline{\omega}^j_{t+1}}\\ &= \frac{\partial \lambda}{\partial \overline{\omega}^j_{t+1}}\left[1 - s^p_{t+1}\left(\Gamma(\overline{\omega}^j_{t+1}) - \mu G(\overline{\omega}^j_{t+1})\right)\right] - \alpha^j k^j_{t+1}\Gamma'(\overline{\omega}^j_{t+1})\left(1-\Gamma(\overline{\omega}^j_{t+1})\right)Var_t[s^p_{t+1}]\\ &= (k^j_{t+1})^{-1}\frac{\partial \lambda}{\partial \overline{\omega}^j_{t+1}} - \alpha^j k^j_{t+1}\Gamma'(\overline{\omega}^j_{t+1})\left(1-\Gamma(\overline{\omega}^j_{t+1})\right)Var_t[s^p_{t+1}].\end{aligned}$$

To complete the evaluation of this term,

$$\frac{\partial\lambda}{\partial\overline{\omega}_{t+1}^{j}} = \mu\frac{\Gamma'(\overline{\omega}_{t+1}^{j})G''(\overline{\omega}_{t+1}^{j}) - \Gamma''(\overline{\omega}_{t+1}^{j})G'(\overline{\omega}_{t+1}^{j})}{\left(\Gamma'(\overline{\omega}_{t+1}^{j}) - \mu G'(\overline{\omega}_{t+1}^{j})\right)}$$
$$+ \alpha^{j}k_{t+1}^{j}\frac{Var_t[s_{t+1}^{p}]}{s_{t+1}^{p}}\left[\frac{\mu\left(1-\Gamma(\overline{\omega}_{t+1}^{j})\right)\left(\Gamma'(\overline{\omega}_{t+1}^{j})G''(\overline{\omega}_{t+1}^{j}) - \Gamma''(\overline{\omega}_{t+1}^{j})G'(\overline{\omega}_{t+1}^{j})\right)}{(\Gamma'(\overline{\omega}_{t+1}^{j}) - \mu G'(\overline{\omega}_{t+1}^{j}))^2}\right]$$
$$- \alpha^{j}k_{t+1}^{j}\frac{Var_t[s_{t+1}^{p}]}{s_{t+1}^{p}}\left[\frac{\Gamma'(\overline{\omega}_{t+1}^{j})^2}{\left(\Gamma'(\overline{\omega}_{t+1}^{j}) - \mu G'(\overline{\omega}_{t+1}^{j})\right)}\right].$$

The result of this analysis is that the sign of $\frac{\partial\lambda - s_{t+1}\Lambda}{\partial\overline{\omega}_{t+1}^{j}}$ is ambiguous, since the sign of $\frac{\partial\lambda}{\partial\overline{\omega}_{t+1}^{j}}$ is ambiguous. In the hedge case, the analogous equation is

$$\frac{\partial\lambda s_{t+1}^{h} - \Lambda}{\partial\overline{\omega}_{t+1}^{j}} = \frac{\mu\left(\Gamma'(\overline{\omega}_{t+1}^{j})G''(\overline{\omega}_{t+1}^{j}) - \Gamma''(\overline{\omega}_{t+1}^{j})G'(\overline{\omega}_{t+1}^{j})\right)}{k_{t+1}^{j}} > 0, \tag{26}$$

the inequality by (10). Inserting this into (21), and with (25) and (26) gives the result that, for hedge firms, k_{t+1}^{j} is positively related to s_{t+1}^{h}. $\frac{\partial\lambda - s_{t+1}^{p}\Lambda}{\overline{\omega}_{t+1}^{j}}$ enters positively in both the numerator and denominator in (20), so the total effect of the magnitude of this term on $\frac{s_{t+1}^{p}}{dk_{t+1}^{j}}$ is not clear.

Finally, the derivatives in (20) involving variance terms will be considered. First,

$$\frac{dVar_t[(1-\Gamma(\overline{\omega}_{t+1}^{j}))s_{t+1}^{p}]}{ds_{t+1}^{p}} = \frac{d}{ds_{t+1}^{p}}\left(E_t\left[(1-\Gamma(\overline{\omega}_{t+1}^{j}))^2 s_{t+1}^{p}\right] - \left(E_t\left[(1-\Gamma(\overline{\omega}_{t+1}^{j}))s_{t+1}^{p}\right]\right)^2\right)$$
$$= 2E_t\left[(1-\Gamma(\overline{\omega}_{t+1}^{j}))s_{t+1}^{p}\left((1-\Gamma(\overline{\omega}_{t+1}^{j})) - \frac{\partial\overline{\omega}_{t+1}^{j}}{\partial s_{t+1}^{p}}s_{t+1}^{p}\Gamma'(\overline{\omega}_{t+1}^{j})\right)\right]$$
$$- 2E_t\left[(1-\Gamma(\overline{\omega}_{t+1}^{j}))s_{t+1}^{p}\right]E_t\left[(1-\Gamma(\overline{\omega}_{t+1}^{j})) - \frac{\partial\overline{\omega}_{t+1}^{j}}{\partial s_{t+1}^{p}}s_{t+1}^{p}\Gamma'(\overline{\omega}_{t+1}^{j})\right],$$

or more succinctly

$$\frac{dVar_t[(1-\Gamma(\overline{\omega}^j_{t+1}))s^p_{t+1}]}{ds^p_{t+1}} = 2Cov_t\left[(1-\Gamma(\overline{\omega}^j_{t+1}))s^p_{t+1}, (1-\Gamma(\overline{\omega}^j_{t+1})) - \frac{\partial\overline{\omega}^j_{t+1}}{\partial s^p_{t+1}}s^p_{t+1}\Gamma'(\overline{\omega}^j_{t+1})\right]. \tag{27}$$

To get a sense of the sign of this covariance term, imagine all terms are at their mean value, and then s^p_{t+1} is positively perturbed above its mean. This will cause $\overline{\omega}^j_{t+1}$ to decrease, by (25), which will cause both $1-\Gamma(\overline{\omega}^j_{t+1})$ and $\Gamma'(\overline{\omega}^j_{t+1})$ to increase. Although (25) also shows that this will cause $\frac{\partial\overline{\omega}^j_{t+1}}{\partial s^p_{t+1}}$, which is negative, to decrease in magnitude, it is very plausible that the overall effect on the second covariance term will be positive.

Similarly,

$$\begin{aligned}
&\frac{d}{dk^j_{t+1}}k^j_{t+1}Var_t\left[(1-\Gamma(\overline{\omega}^j_{t+1}))s^p_{t+1}\right] \\
&= Var_t\left[(1-\Gamma(\overline{\omega}^j_{t+1}))s^p_{t+1}\right] + \frac{\partial\overline{\omega}^j_{t+1}}{\partial k^j_{t+1}}\left(E_t\left[(1-\Gamma(\overline{\omega}^j_{t+1}))^2 s^{p2}_{t+1}\right] - \left(E_t\left[(1-\Gamma(\overline{\omega}^j_{t+1}))s^p_{t+1}\right]\right)^2\right) \\
&= Var_t\left[(1-\Gamma(\overline{\omega}^j_{t+1}))s^p_{t+1}\right] - 2\frac{\partial\overline{\omega}^j_{t+1}}{\partial k^j_{t+1}}k^j_{t+1}Cov_t\left[(1-\Gamma(\overline{\omega}^j_{t+1}))s^p_{t+1}, \Gamma'(\overline{\omega}^j_{t+1})s^p_{t+1}\right].
\end{aligned}$$

With (26), this is

$$\frac{d}{dk^j_{t+1}}k^j_{t+1}Var_t\left[(1-\Gamma(\overline{\omega}^j_{t+1}))s^p_{t+1}\right] \tag{28}$$

$$= Var_t\left[(1-\Gamma(\overline{\omega}^j_{t+1}))s^p_{t+1}\right] - 2\frac{Cov_t\left[(1-\Gamma(\overline{\omega}^j_{t+1}))s^p_{t+1}, \Gamma'(\overline{\omega}^j_{t+1})s^p_{t+1}\right]}{(\Gamma'(\overline{\omega}^j_{t+1}) - \mu G'(\overline{\omega}^j_{t+1}))k^j_{t+1}s^p_{t+1}}. \tag{29}$$

As can be shown using the same argument as above, the covariance term in (30) is positive, hence (30) might be negative. Moreover, during an expansionary shock, the both k^j_{t+1} and s^p_{t+1} will decrease, increasing the negative term in (30) and, by (28), potentially decreasing the positive term. Hence, since (29) enters negatively in the numerator of (20), during an expansion equations (28) and (30) could very well act to increase the slope $\frac{ds^p_{t+1}}{dk^j_{t+1}}$, either increasing the augmenting effect on the financial accelerator of the existence of Ponzi firms, or bringing the effect closer to being augmenting, depending on the ambiguities of the rest of the terms in (20). If the incorporation of Ponzi firms in this model is to increase the effect of the financial accelerator, it is through equations (28) and (29)/(30).

4.2.2 Firm-mode Decision Rule

For ease, (11) is restated below

$$\alpha^*_{t+1} \equiv \frac{2N^j_t E_{t-1}[(1-\Gamma_h(\overline{\omega}^j_t))s^h_t - (1-\Gamma_p(\overline{\omega}^j_t))s^p_t]}{K^j_t Q_{t-1} Var_t[(1-\Gamma_p(\overline{\omega}^j_t))s^p_t]}$$

α^*_{t+1} is increasing in net worth and decreasing in asset prices, the level of capital, and the variance $Var_{t-1}\left[(1-\Gamma(\overline{\omega}^j_{t+1}))s^p_t\right]$. Since, by (12), ϕ_{t+1} is decreasing in α^*_{t+1}, the proportion of firms that are Ponzi is decreasing in net worth and increasing in the level of capital, in asset prices, and the variance term above. During expansions, the increase in net worth puts negative pressure on ϕ_{t+1}, but the increases in K_t and Q_{t+1} push ϕ_{t+1} upward. By (28) and (29/30), the decreases in the return ratio and the capital-equity ratio might push the variance term in opposite directions. The overall effect is therefore ambiguous.

To state the results of the model sketch concisely, the conditions under which the inclusion of Ponzi firms would augment the accelerator are: i) the relationship between the return ratio and the capital-equity ratio is stronger for Ponzi firms than for hedge firms, $\frac{ds^p_{t+1}}{d^j_{k_{t+1}}} > \frac{ds^h_{t+1}}{d^j_{k_{t+1}}}$, and $\frac{ds^p_{t+1}}{d^j_{k_{t+1}}}$ increases during expansion; ii) the proportion of Ponzi firms increases during expansions. Both of these conditions are possible, with the driving force being how the variance terms in the model respond to expansions, captured in equations (28) and (29/30).

5 Conclusion

The clear next step for the model in section 4 would be to linearize, calibrate, and evaluate the model numerically. This is crucial in order to understand the implications of a financial system wherein a subset of the borrowers are risk-loving, and the size of this subset evolves based on the state of the economy; the analytical evaluation done here is ambiguous.

It should also be noted that it is an empirical question whether or not firms, and economic agents generally, have preferences toward risk that are dynamic based on the current and recent states of the economy, as in Minsky, or whether such preferences are static, as in the DSGE models. Hence it is an open question whether or not it is worthwhile to do what is attempted here incorporate this Minskain feature into macroeconomic models. Thus another avenue for future research would be to test whether firms take on a higher capital-equity ratio and engage in more behavior that could be characterized as risk-loving in expansionary phases of the business cycle than in recessionary phases. A similar empirical question is how agents update subjective probabilities over different phases of the business cycle, and whether this can be characterized by path-dependence. This is not a feature that could fit into the DSGE style, however, since agents in DSGE models always have rational expectations and subject distributions that match objective probability distributions.

Other features of the FIH highlighted in section 2 are less empirical. The balance sheets, wealth, and debt of firms and households clearly matter in the real economy, as evidenced by the Great Recession. Intertemporal linkages in capital financing and in many other kinds of contracts and long term decisions also clearly matter in the actual economy. That the validation of agreements and decisions made at a particular time depend upon how events will play out in the uncertain future is clearly a source of instability in the economy, and models that abstract away from this fact rule out this instability and volatility by assumption. Hence, out of the FIH, there are some claims and hypotheses that may turn out to have tenuous connection with reality and be unimportant. However, there are others which seem crucial to understanding the actual operation of the economy, and which, if incorporated into theory, will lead to more valid models.

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