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Holism and non-separability applied to quantum mechanics

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Holism and Non-separability Applied to Quantum Mechanics

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Dedication

This thesis is dedicated to three parents and an older brother whose support, encouragement, and

love have given me the strength to pursue all of my dreams.

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Acknowledgements

I would like to thank Dr. Tracy Lupher for allowing me the opportunity to work with him as my thesis advisor. Thank you, Dr. Lupher, for not giving up on me when my ambitions may have seemed too large. I have appreciated all of the insight you've given me. I would also like to thank Dr. Dorn Peterson and Dr. William Ingham for serving as my readers. Your words of praise and encouragement will stick with me.

Abstract

Einstein was never satisfied with quantum mechanics. He argued that quantum mechanics was incomplete for two main reasons; it violated the locality principle and the separability principle. The violation of separability is an unavoidable consequence of quantum interactions. Non-separability can be seen in quantum entanglement. Non-locality, however, is more controversial. Einstein and his associates published the EPR paper in order to argue for the incompleteness of quantum mechanics. Years later, John Bell formulated what became known as the Bell Inequalities in response to the EPR paper. The Bell Inequalities are seen as a major obstacle for quantum locality. I will argue that non-locality is not a necessary implication of the Bell Inequalities. The Bell Inequalities were developed using Bell's locality requirement as a major premise. Bell's locality requirement can be described in terms of two conditions, parameter independence and outcome independence. A violation of either condition will lead to a violation of the Bell inequalities. Parameter independence is not violated by the results of experimental quantum physics. So, it can be argued that violations of the Bell inequalities are caused by the violation of outcome independence. Such a violation of outcome independence does not imply non-locality if we accept some form of holism or non-separability. Thus, by including some form of holism or non-separability into our picture of the quantum realm we can develop a theory that does not conflict with locality. This paper will discuss different types of holism and non-separability and how they can be used to help understand quantum phenomena.

I. Introduction

I.1 Introduction to Problem

The twentieth century brought the development of a new description of physical reality:

quantum mechanics. Quantum mechanics is a mathematical formulation used for understanding

and making predictions about microscopic systems. Quantum mechanics applies many of the

same concepts used in classical mechanics to the microscopic realm. Quantum entities, however,

do not always perfectly follow the laws of classical physics. For example, the precise trajectory

for a particle is unsustainable. In a somewhat similar manner to classical physics, quantum

mechanics describes systems as being in states. Physical observables, such as momentum,

position, energy, etc, are used to describe the state of a system. For example, an atom can possess

a finite number of energy levels, each of these energy levels can be thought of as a different

energy state. In order to gather information about a quantum system's state physicists utilize a

mathematical description called the wavefunction. The wavefunction of a system describes the

system as it is evolved by the Schrödinger equation[1]. A particle's wavefunction is spread out in

space due to the particle's wave-like nature[2]. The lack of localization of the particle that is given

by the wavefunction is one of quantum mechanics' major deviations from classical mechanics. In

classical mechanics the position of the elements of a system can be calculated. However, in

quantum mechanics the description of the elements of a system are given by the wavefunction,

which spans over some region of space. Often times we deal with the wavefunction ranging over

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[1] The Schrödinger equation was developed by Austrian physicist Erwin Schrödinger. It uses conservation of energy to predict the behavior of a system. Given initial conditions $\Psi(x,0)$, the Schrodinger equation determines $\Psi(x,t)$ for all future times (Griffiths 2005).

[2] Louis De Broglie (1923) theorized that matter, similarly to light, had both wave-like and particle-like properties.

infinite space. In order to gain meaningful information about the position of a particle we use

Born's statistical interpretation[3]. We can also use the wavefunction to gain probabilistic

information about other physical observables that pertain to the state of the system. So in

quantum mechanics we see an element of indeterminacy, whereas in classical mechanics[4] there is

not any indeterminacy. This factor of indeterminacy alone has raised issues with physicists and

philosophers. Some take a realist approach. A scientific realist believes that if an element of

physical reality exists then it should be able to be described by theory. Realists argue that a

particle has to have a position at all times and quantum mechanics should therefore, always be

able to identify its position. Quantum mechanics cannot identify a particle's position at all times,

so realists argue that quantum mechanics is incomplete. Albert Einstein and his colleagues, Boris

Podolsky and Nathan Rosen, held this belief. The alternative view was expressed by the

Copenhagen interpretation. According to this account quantum mechanics is complete. The

Copenhagen interpretation claims that it is not possible to simultaneously know all of the values

for all of the properties of a system. The best knowledge that we can have of these unknown

properties is probabilistic. This is the most widely accepted interpretation of quantum mechanics

by physicists.

Despite its indeterminist nature, quantum mechanics has been extremely useful in helping

us gain insight into the nature of quantum particles and their interactions with one another. We

have learned that quantum interactions are not as easily described as interactions seen in classical

physics. In quantum mechanics interacting particles' wavefunctions cannot be written as single

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[3] Born's Statistical Interpretation tells us that for quantum particles the probability of finding a particle in a region of space is the integral of the complex square of the wavefunction. This is given by $\int |\Psi(x,t)|^2 \, dx$.

[4] Classical mechanics in this sense excludes all statistical mechanics.

wavefunction, they must be written as products. This means that the states of separate systems are combined to form a new state. Such a superposition of states could not exist in classical mechanics. One curious effect that deals with quantum interactions that will be discussed throughout this paper is quantum *entanglement*. In quantum entanglement two interacting systems are seen as losing their individuality. When individual systems come into interaction with one another we see a superposition of their states. Once they are interacting, their states become indistinguishable, or impossible to separate once again. This evidence of *non-separability* in quantum interactions has raised some question about the nature of reality. In quantum mechanics we cannot always distinguish separate systems. To make sense of these effects we can imagine that some type of *holism* may be at work on the quantum level. Holism is a theory that claims that a system cannot be explained in terms of its parts. Essentially, it states that a system is more than the sum of its parts. In this paper, I will give a further introduction to the concepts of holism and non-separability and how they are seen in quantum entanglement. I will then take a look at the history of the argument, as originally given by Einstein, that quantum mechanics is incomplete. Einstein's arguments for the shortcomings of quantum mechanics are based on his beliefs that [1]systems should be separable and [2]systems should not communicate in a *non-local* manner. Non-locality was something that Einstein could not accept because the notions of quantum non-locality conflict with special relativity. Einstein's arguments were made famous with the publication of the Einstein-Podolsky-Rosen paper. John Bell gave a counterargument to the EPR paper, which became known as the Bell Inequalities. The Bell Inequalities suggest that quantum systems do act in a *non-local* manner. In this paper, I will argue that if some type of non-separability or holism is operating at the quantum level then

conflicts between quantum mechanics and special relativity disappear. Thus, by including non-separability or holism in quantum mechanics we can dismiss notions of non-locality.

I.2 Outline of Thesis

In section II of this paper, I will give an introduction to the concepts of holism and non-separability. These concepts will be important throughout this paper as I will examine how they can be applied to quantum mechanics. I will argue that by including holism or non-separability in our understanding of quantum mechanics we can eliminate tensions between Einstein's arguments for the incompleteness of quantum mechanics and the Bell Inequalities. The concepts of holism and non-separability are most clearly seen in quantum entanglement. In section III of this paper, I will discuss quantum entanglement. I will also give an example of how non-separability can be seen in the mathematical description of entangled systems.

I will then begin to discuss Einstein's arguments for the incompleteness of quantum mechanics. In section IV.1, I will introduce the problems Einstein saw in quantum mechanics in terms of separability. In section IV.2 I will then discuss how quantum mechanics conflicts with special relativity. This will include a discussion of the concept of non-locality. I will also discuss how Einstein used hidden variables to repair the deficiencies he saw in quantum mechanics.

In section V, I will discuss the Einstein-Podolsky-Rosen paper, which expresses these physicsts' criticisms of the completeness of quantum mechanics. Although it was Einstein's ideas that spawned the EPR paper, Podolsky was the author of the article. After publication of the EPR paper Einstein made it clear that his theories were not accurately presented. In section V.2 I will give a summary of what shortcomings Einstein saw in the EPR paper and what he had intended to communicate. Lastly in section V.3, I will discuss a version of the thought

experiment proposed in the EPR paper given by David Bohm. Bohm's version of the EPR experiment is important because it allows a more practical way to experimentally test the EPR conjectures.

In response to the EPR paper, John Bell derived the famous Bell inequalities. I will discuss the Bell Inequalities in section VI. The Bell inequalities are said to put the arguments proposed in the EPR paper to rest. In doing so, the Bell inequalities seem to suggest that quantum mechanics is in conflict with special relativity. In section VI.1 I will discuss the Bell locality requirement, which Bell's entire argument rests upon. In sections VI.2, VI.4, and VI.5 I will review various versions of the Bell Inequality.

Section VII will be dedicated to this claim that Bell's locality requirement possesses factorizability. According to philosophers Abner Shimony (2009) and Jon Jarrett (1984), Bell's locality requirement can be factorized into two conditions, parameter independence and outcome independence. In section VII.1 I will discuss what is meant by parameter and outcome independence. In section VII.2 I will then discuss the implications of violating either condition. A violation of either condition will lead to a violation of the Bell inequalities. It has been seen that the condition for parameter independence is not violated experimentally. Jon Jarrett (1984) and Abner Shimony (2009) make a compelling case that experimental violations of the Bell inequalities (Aspect, Dalibard 1982) are caused by the violation of outcome independence. Such a violation of outcome independence does not directly imply non-locality if we accept some form of holism or non-separability. In section VIII, I will discuss the different forms of holism and non-separability that have been discussed in relation to quantum mechanics. I will discuss how these forms of holism and non-separability could be applied.

II. Introduction to Holism and Non-separability

Interactions between quantum systems are often described as being "entangled." Such interactions involve individual constituents of a compound system losing their individuality and becoming part of a single system. Entangled particles do not possess subjective properties. Giancarlo Ghirardi (2005) describes this view of an entangled quantum system as "an unbroken whole" or "an undivided unity." Such a theory provides a holistic or non-separable view of quantum interactions.

Holism and non-separability are concepts that dismiss the significance of single parts in a compound system. Holism describes the compound system in terms of the nature of the whole system. Non-separability describes the compound system in terms of the interactions between the parts of the system.

Holism: the sum of a system is more than its parts. In other words, for something to be whole it is not enough for just the sum of its parts to exist. Holism gives leeway for the state of a system to be determined by something more than the state of its parts.

Non-separability: the components of a compound system do not possess a distinct individuality. It makes the claim that a physical system is not completely determined by the physical interactions and relations between its parts.

Loosely speaking, holism can be thought of in terms of a single entity whose parts alone are not what define its identity. Non-separability can be thought of in terms an entity composed of inseparable parts whose parts have indistinguishable identities.

III. Entanglement

While discussing holism and non-separability a brief explanation of entangled systems was given. Entanglement offers evidence of quantum entities behaving in a holistic or non-separable manner. Section III.1 will be used to elaborate on the theory of quantum entanglement. Mathematical evidence of non-separability occurring during quantum entanglement will then be given in section III.2.

Section III.1 Entanglement in Quantum Mechanics

In Bernard D'Espagnat's *Veiled Reality* (2003) he introduces the concept of entanglement by first revisiting the approach classical physics takes to understanding nature. He explains that classical physics describes nature by dividing extended physical systems into parts. This system of thought allows us to think in terms of localized elements, or parts, that interact through the forces. He defines classical physics as "divisibility by thought." To D'Espagnat "divisibility by thought" is classical physics' description of extended physical systems as being composed of parts localized in different regions of space. Through having a complete knowledge of the values of physical quantities pertaining to each of these constituent parts one is said to have a complete knowledge of the composite system. D'Espagnat argues that quantum mechanics disproves this understanding of nature as being able to be carved into parts. In quantum mechanics we can have a system, let us denote it **Z**, which is composed of two physical systems X and Y. Systems X and Y are interacting. We can then imagine that at some time, t_0, the system **Z** is describable in terms of the product wavefunction of the X and Y. However, as the system **Z** evolves to times greater than t_0, the wavefunction of **Z** will no longer be able to be separated into the systems X and Y. So, the wavefunction of **Z**, which began as the product of separate systems X and Y, evolved in such a way that X and Y are no longer separable systems.

This means that a complete knowledge of the wavefunction of the individual systems, X and Y, cannot be known at times greater than t_0. This effect has been described by Schrödinger as "entanglement." Systems that are entangled are in mathematically non-separable states.

Section III.2 A Mathematical Example of Entanglement Using Spin

In this section, we will take a look at the mathematics used to describe entangled systems. To do so, we will have to take the tensor product of two state vectors. We can write two state vectors, A and B, as follows.

$$| A > = \begin{pmatrix} a \\ b \end{pmatrix} \qquad | B > = \begin{pmatrix} c \\ d \end{pmatrix}$$

These states will yield a tensor product of,

$$| A > \otimes | B > = \begin{pmatrix} a*c \\ a*d \\ b*c \\ b*d \end{pmatrix}$$

Now, that we have seen the basic rule for tensor products we can use it in the following discussion on quantum entanglement.

Quantum entanglement can be demonstrated mathematically using spin states. Spin states of particles can be described in terms of Pauli spin matrices. Pauli spin matrices describe the orientation of spin, up or down, along a particular axis, the x-axis, y-axis, or z- axis. Pauli spin matrices consist of two eigenvalues, either 1 or -1. The Pauli matrices for the spin along the z-axis have the following eigenvectors,

Spin-up: $| up > = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Spin-down : $| down > = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Let us now imagine the singlet-state for a pair of entangled particles, with one in a spin-up state and the other in a spin-down state. The wavefunction for a singlet-state is given by,

$$| \Psi > = 1/\sqrt{2} \, (| \varphi_1 > \otimes | \varphi_2 > - | \varphi_2 > \otimes | \varphi_1 >).$$

Applying the spin states,

$$| \Psi > = 1/\sqrt{2} \, (| up > \otimes | down > - | down > \otimes | up >)$$

where ,

$$| up > \otimes | down > = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1*0 \\ 1*1 \\ 0*0 \\ 0*1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

and,

$$| down > \otimes | up > = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0*1 \\ 0*0 \\ 1*1 \\ 1*0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

So,

$$| \Psi > = \quad (1/\sqrt{2}) * \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right) = (1/\sqrt{2}) * \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$$

Above we can see the combined state of the two systems. To see how this state has become non-separable we can try to separate the system back into two separate systems. For a system with the wavefunction, Ψ, to be separable it must be able to be divided into the product of the states of two systems, φ_1 and φ_2. The following equation is known as the separability condition.

$$| \Psi >= | \varphi_1 > \otimes | \varphi_2 >$$

Let the two unknown vectors be represented by A and B. In order to satisfy the separability condition, we should let the product of our unknown states A and B be equal to the wavefunction we found for our singlet state.

$$| A > \otimes | B > = | \Psi >$$

$$\begin{pmatrix} a*c \\ a*d \\ b*c \\ b*d \end{pmatrix} = (1/\sqrt{2}) * \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$$

From this we derive the following set of equations,

$$a*c = (1/\sqrt{2}) *0 = 0$$

$$a*d = (1/\sqrt{2}) *1 = 1/\sqrt{2}$$

$$b*c = (1/\sqrt{2}) *-1 = -1/\sqrt{2}$$

$$b*d = (1/\sqrt{2}) *0 = 0$$

If we then try to solve these equations for their unknowns we find that it is impossible. If $a*c =$ 0, then we know either a or c must be equal to zero. However, $a*d \neq 0$ so a cannot be equal to zero and $b*c \neq 0$ so c cannot be equal to zero. This means we cannot satisfy the separability condition.

IV. Einstein and Quantum Mechanics

We have now looked at the basic concepts of holism, non-separability, and quantum entanglement. These concepts will become important once we examine the debate concerning the completeness of quantum mechanics. This debate was triggered by Einstein's personal dissatisfaction with quantum mechanics. Einstein was dissatisfied with quantum mechanics for a number of reasons. First, Einstein saw the statistical nature of quantum mechanics as problematic. In his letters to Born (Einstein 1926), Einstein is famously quoted as saying, "I am convinced that He (God) is not playing at dice." This quote represents Einstein's view that there could be a way to describe quantum mechanics by a deterministic theory, rather than a statistical one Ballentine (1972, p. 1763). Although this aspect of Einstein's argument against the completeness of quantum mechanics is not prudent to the argument in this paper, it helps to illustrate Einstein's scientific realism. Einstein believed that if an element of physical reality exists then scientific theory should be able to predict that element of physical reality with certainty. This stance also caused Einstein to argue that quantum mechanics should allow for the spatial separability of systems, this argument will be discussed in terms of *Einstein's separability principle*. He also argued that quantum mechanics conflicted with special relativity, this argument will be discussed in terms of the *locality principle*. In section IV.1, I will first discuss the separability principle. In section IV.2, I will discuss the locality principle.

Section IV.1 Einstein Separability

Einstein believed that quantum mechanics failed to provide a description of each individual element of reality. He believed that the wavefunction was not able to provide a description of an individual system, but rather an ensemble of similar systems Ballentine (1972,

p. 1763). Einstein argued that if the wavefunction cannot provide a full description of an individual system then, we cannot extract information about the physical properties that describe individual systems, such as position and momentum. Thus, Einstein believed that quantum mechanics does not give conclusive evidence of how an individual system behaves in nature. This problem caused issues to the realism provided by quantum mechanics. According to Einstein, a complete scientific theory should be able to provide a physical quantity corresponding to every element of physical reality. Thus, critical to Einstein's arguments of the incompleteness of quantum mechanics is his separability principle.

> *Einstein separability*: two spatially separated systems possess their own unique real states.

Section IV.2 Non-locality and Hidden Variables

One of the laws given by special relativity is that nothing can travel faster than the speed of light. We will call this Einstein locality.

> *Einstein locality*: all physical effects are propagated at speeds that do not exceed the speed of light.

Non-locality is the denial of the locality principle. Often times it is argued that quantum mechanics involves non-locality. How exactly non-locality is seen in quantum mechanics will be discussed in section V.3 on Bohm's version of the EPR experiment. Einstein and his colleagues believed that non-locality did not exist at the quantum scale. They attributed the appearance of non-locality in quantum mechanics to the incompleteness of the theory. In order to account for our lack of understanding of the inner workings of the quantum realm, we can insert hidden

variables into quantum mechanics. The physical preparation of the system and its evolution are said to bring about a distribution of hidden variables that are unknown to us. Hidden variables are parameters that completely specify the state of the system. In this way hidden variables give definite information about what state a system is in (Ghirardi 2005, p. 195). For a discrete set of values that an observable quantity can have, hidden variables only allow the system to possess one of these values. For example, consider again an atom with a discrete number of energy states that it is allowed to be in. By inserting hidden variables we see that the value of the atom's energy will be precisely determined (Ghirardi 2005, p. 196).

Section IV.3 What Should Be Taken from Einstein's Criticisms?

Einstein's argument suggests that in order for quantum mechanics to be complete it must obey the separability principle and the locality principle (Howard 1985, p. 173). However, in section III on quantum entanglement we have seen that non-separability must be included in quantum mechanics. Entanglement is not a result of how quantum mechanics is applied to a system composed of interacting particles; rather entanglement is a real quality of the interactions between those particles. Thus, Einstein's argument that quantum mechanics should follow the separability principle is inconsistent with experimental evidence. On the other hand, Einstein's argument that a violation of non-locality would conflict with special relativity is of great significance. In this paper, I will argue that by accepting non-separability or holism quantum mechanics does not violate non-locality. So, we should neglect Einstein's criticisms of non-separability and we should acknowledge his criticisms of non-locality.

V. The EPR Paper

Einstein's arguments for the incompleteness of quantum mechanics became most famous through the publication of the Einstein-Podolsky-Rosen paper. In 1935 Einstein, Podolsky, and Rosen published an article entitled "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" in the *Physical Review*. The EPR paper's essential argument was that quantum mechanics is incomplete. However, the EPR paper was written by Podolsky and it is believed that Einstein never even saw the final draft before its submission to the *Physical Review* (Howard 1985). In a letter to Schrödinger, Einstein expresses his grievances with the EPR paper as he laments that the essential point was lost in formalism. Later in his letter to Schrödinger, Einstein presents his own arguments for the incompleteness of quantum mechanics. In considering the EPR paper it is important to consider both the ideas conveyed by Podolsky in the EPR paper and Einstein's actual arguments. In section IV.1, I will discuss the arguments that were given in the EPR paper. This will be followed by Einstein's criticisms of the EPR paper and the argument he intended to give in the EPR paper in section IV.2. Lastly, in section IV.3, I will give a version of the thought experiment proposed in the EPR paper that was devised by David Bohm. Bohm's version of the EPR experiment is helpful in demonstrating how Einstein saw quantum mechanics as implying non-locality. In section IV.3, I will also discuss how Einstein locality is seen as being violated in quantum mechanics in relation to Bohm's thought experiment.

Section IV.1 The Argument Given in EPR

The main argument for the incompleteness of quantum mechanics given by the EPR paper concerns the inability to know simultaneously and with certainty the values of physical

quantities represented by non-commuting operators. This means that although we can know that

a particle possesses multiple properties simultaneously, for example that a particle has both spin

along the x-axis and y-axis, we cannot know quantitatively what these properties are

simultaneously. Quantum mechanics does not allow us to know such properties simultaneously

because their operators do not commute. The EPR paper makes it clear that they will only be

addressing the question of, "Is quantum mechanics complete?" and not the question, "Is quantum

mechanics correct?" (Einstein, Podolsky, Rosen 1935, p. 777). The EPR paper sets out by stating

two conditions. First, it establishes a condition for a physical theory to be considered complete.

This condition requires that every element of physical reality must have a corresponding

counterpart in the physical theory. Second, it establishes a criterion for determining an element

of physical reality or the "Criterion for Reality." This criterion requires that if a physical quantity

can be predicted with certainty, without disturbing the system, then there exists an element of

physical reality corresponding to this physical quantity (Einstein, Podolsky, Rosen 1935, p. 777).

To then prove the incompleteness of quantum mechanics the paper attempts to show a

case in which we can know the simultaneous realities of two non-communicating operators. The

paper considers two particles that were at one time interacting, but are no longer interacting

either directly or indirectly. If quantum mechanics is correct, then the wavefunction used to

describe the two-particle system is considered to be an eigenfunction of the difference in the

position operators, where $q_2 - q_1 = x_0$, and the sum of the momentum operators, where $p_2 + p_1 =$

0 (Ballentine 1972, p. 1766). This can be proven as the difference in position operators

commutes with the sum of the momentum operators. Now, if the position of the first particle is

measured to be $q_1 = x$, then we can conclude that the position of the second particle must be $q_2 =$

$x + x_0$. Thus, without disturbing the system in any way we have obtained physical quantities for

the positions of both particles with certainty. According to the Criterion for Reality, we can conclude that there exists a part of reality corresponding to these physical quantities. We can also measure the momentum of particle 1 with certainty. By similar argumentation we can conclude that we can that find a physical quantity corresponding to the momentum of particle 2. Thus, we can say that both p_1 and p_2 are elements of physical reality (Ballentine 1972, p. 1766).

Having set up this scenario, which allows us to know the simultaneous reality of the position and momentum of both particles, we can conclude quantum mechanics is incomplete. According to quantum mechanics we cannot know with certainty the simultaneous values of two non-commuting operators. However, we have just shown that in the above instance we can in fact know with absolute certainty the value of two non-commuting operators. Thus, quantum mechanics does not fulfill the first condition set out by the EPR paper, the condition for a physical theory to be considered complete. The incompleteness of quantum mechanics is attributed to the wavefunction's description of the system because no wavefunction can be an eigenfunction of both q_2 or p_2 simultaneously.

Section IV.2 Einstein's Own Argument

In Einstein's own argument he does not discuss the "Criterion for Reality." Einstein also makes it evident that he is not concerned with the simultaneous values for incompatible quantities. Einstein is instead concerned with the description quantum mechanics offers of a single quantity, like momentum or position. Einstein focuses on the inability to assert locality and separability, on one hand, and the completeness in the description of individual systems by means of the wavefunction, on the other hand. The "paradox" seen by Einstein is that both of these conditions can never be true at the same time (Fine 2009).

One version of Einstein's argument, which best illustrates this dilemma, can be seen in his letter to Schrödinger. He imagines the interaction between two systems, let them be called the Albert and Niels systems, where their relative positions are constant. He then asks the reader to imagine the composite system, the Albert+Niels system, when the two systems are spatially separated (Fine 2009). If we assume the principles of locality and separability, then whether a physical property holds for Niels' system does not depend on what measurements are made on Albert's system. If we know the relative positions of the systems are constant, we can measure the position of Albert's system, and therefore deduce the position of Niels' system. By assuming a principle of locality-separability we can conclude that Niels' system must already have a determinate position immediately before the measurement is taken on Albert's system. However, at that time we have no independent wavefunction to describe Niels' system. The only description we have is of the total system, Albert+Niels. This total wavefunction is unable to predict with certainty the position of Niels' system. Einstein argues that the description of Niels' system as provided by the wavefunction is then incomplete; it cannot give definite information about the true physical properties of Niels' system. In Einstein's formulation we see the locality-separability and eigenvalue-eigenstate connection are in conflict (Fine 2009). A physical quantity of a system will have an eigenvalue if and only if the state of the system is an eigenstate, or a mixture of eigenstates, of that quantity with that eigenvalue. If we then have a composite system, such as the Albert+Niels system, in an eigenstate with a corresponding eigenvalue we can only know the eigenvalue for the total system. Thus, we see a violation to the locality-separability principle. The problem is not seen as a violation of locality, rather the issue is with separability as eigenvalues for eigenstates cannot be recovered for the individual systems.

Section IV.3 Bohm's Version Using Spin

In 1951 Bohm wrote an article entitled "A Suggested Interpretation of the Quantum

Theory in Terms of "Hidden" Variables", in which he developed another version of the thought

experiment proposed in the EPR paper. Bohm showed how the thought experiment could be

modeled using a decaying diatomic molecule, whose total spin angular momentum always equals

zero (Fine 2009). When the atoms in the molecule decay they move freely away from each other

in opposite directions. Let us call the place that they are emitted, the source. Detectors may be

placed in the directions that the atoms are emitted to measure their spin components, see figure 1

below. In subsequent measurements it is always found that there is an anti-correlation between

the spin component measurements of the two atoms. This means that if one of the atoms from the

dissociated molecule were measured to have spin up in a particular orientation, then the other

atom would be measured to have spin down with respect to this orientation. For example, if the

detector A in the figure below measures the atom to have spin up along the z-axis then detector B

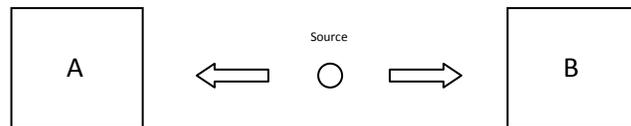will measure the other atom to have spin down along the z-axis.

Figure 1: Experimental setup of source particle that will decay into 2 particles moving

in opposite directions towards detectors at points A and B.

In Bohm's version of the EPR experiment, we see an example of two distinct spatially

separated systems, where we can consider each atom heading towards its respective detector, to

be a system. According to Einstein locality these two separated systems, must not be able to

communicate with each other faster than the speed of light. Let us call one of the systems S_1 and the other S_2. Einstein's locality requirement claims that the real state of S_2 should not be able to influence the real state of system, S_1. However, experimental evidence shows that the states of the system are always anti-correlated. Thus, there appears to be a violation to Einstein locality. To further explore non-locality in quantum mechanics we will now examine Bell's Inequalities.

VI. Bell's Inequalities

Thirty years after the EPR paper was written John Stewart Bell formulated his famous Bell Inequality. This inequality is seen to suggest that there can be no local hidden variable theory that reproduces all of the same results as quantum mechanics. The Bell Inequality has been reproduced in various different ways. Each version of the Bell Inequality begins with the Bell locality requirement as the major assumption. In this section I will first give a derivation of Bell's Inequality based on a derivation by Giancarlo Ghirardi. I will then show two simpler ways to derive the Bell Inequality, first using spin and second using minimal assumptions, taken from Marc Lange's *An Introduction to the Philosophy of Physics: Locality, Fields, Energy and Mass*.

Section VI.1 Bell's Locality Requirement

Bell's Locality requirement makes a general statement about the probability of obtaining a pair of results in a joint measurement. The probability of a joint measurement is said to equal to the product of the probability of obtaining each independent measurement.

$$p_\lambda^{AB} (\mathbf{a},\mathbf{b};\, \alpha,\beta) = p_\lambda^{A}(\mathbf{a},*;\, \alpha) \text{ x } p_\lambda^{B}(*,\mathbf{b};\, \beta) \qquad\qquad (\text{Eq 1})$$

In Bell's Locality requirement we take into account all aspects the physical state of the system including both variables explicit and hidden. We allow this all-inclusive description of the state of a physical system to be represented by λ. Measurements of two observables, $\mathbf{a}$ and $\mathbf{b}$, will be taken of the system at opposite sides of the experimental setup denoted by regions A and B (see Figure 1), yielding results α and β. By $p_\lambda^{AB} (\mathbf{a},\mathbf{b};\, \alpha,\beta)$ we represent the probability of obtaining α and β when measuring $\mathbf{a}$ and $\mathbf{b}$ at regions A and B. By $p_\lambda^{A}(\mathbf{a},*;\, \alpha)$ we represent the probability of

obtaining a measurement α for **a** in region A. Similarly, by $p_\lambda^B(*,\mathbf{b}; \beta)$ we represent the probability of obtaining β for **b** in region B.

Section VI.2 Proof of Bell's Inequality by Giancarlo Ghirardi

The following proof has modified from Giancarlo Ghirardi's *Sneaking a Look at God's Cards*.

We can define $E_\lambda(\mathbf{a},\mathbf{b})$ as the sum of the probabilities of obtaining consistent results, the same measurements of α and β, minus the results of obtaining dissimilar measurements

$$E_\lambda(\mathbf{a},\mathbf{b}) = p_\lambda^{AB}(\mathbf{a},\mathbf{b}; \text{yes, yes}) - p_\lambda^{AB}(\mathbf{a},\mathbf{b}; \text{yes, no}) - p_\lambda^{AB}(\mathbf{a},\mathbf{b}; \text{no, yes}) + p_\lambda^{AB}(\mathbf{a},\mathbf{b}; \text{no, no}). \quad \text{(Eq 2)}$$

In the above equation for $E_\lambda(\mathbf{a},\mathbf{b})$ we allow the outcomes of α and β to be limited to two possibilities, yes or no. We can imagine that these yes or no outcomes refer to whether or not a photon passes a test for polarization. Substituting Bell's Locality requirement into $E_\lambda(\mathbf{a},\mathbf{b})$ we find

$$E_\lambda(\mathbf{a},\mathbf{b}) = [p_\lambda^A(\mathbf{a},*; \text{yes}) \times p_\lambda^B(*,\mathbf{b}; \text{yes})] - [p_\lambda^A(\mathbf{a},*; \text{yes}) \times p_\lambda^B(*,\mathbf{b}; \text{no})] \quad \text{(Eq3)}$$

$$- [p_\lambda^A(\mathbf{a},*; \text{no}) \times p_\lambda^B(*,\mathbf{b}; \text{yes})] + [p_\lambda^A(\mathbf{a},*; \text{no}) \times p_\lambda^B(*,\mathbf{b}; \text{no})]$$

which, can be factored to obtain

$$E_\lambda(\mathbf{a},\mathbf{b}) = [p_\lambda^A(\mathbf{a},*; \text{yes}) - p_\lambda^B(*,\mathbf{b}; \text{no})] \times [p_\lambda^A(\mathbf{a},*; \text{yes}) - p_\lambda^B(*,\mathbf{b}; \text{no})]. \quad \text{(Eq 4)}$$

We can write an equation for a measurement of **d** similar to that of Equation 3,

$$E_\lambda(\mathbf{a},\mathbf{d}) = [p_\lambda^A(\mathbf{a},*; \text{yes}) \times p_\lambda^B(*,\mathbf{d}; \text{yes})] - [p_\lambda^A(\mathbf{a},*; \text{yes}) \times p_\lambda^B(*,\mathbf{d}; \text{no})] \quad \text{(Eq 5)}$$

$$- [p_\lambda^A(\mathbf{a},*; \text{no}) \times p_\lambda^B(*,\mathbf{d}; \text{yes})] + [p_\lambda^A(\mathbf{a},*; \text{no}) \times p_\lambda^B(*,\mathbf{d}; \text{no})].$$

If we subtract $E_\lambda(\mathbf{a},\mathbf{d})$ from $E_\lambda(\mathbf{a},\mathbf{b})$ we have

$E_\lambda(\mathbf{a},\mathbf{b}) - E_\lambda(\mathbf{a},\mathbf{d}) = \{[p_\lambda{}^A(\mathbf{a},*; \text{yes}) \times p_\lambda{}^B(*,\mathbf{b}; \text{yes})] - [p_\lambda{}^A(\mathbf{a},*; \text{yes}) \times p_\lambda{}^B(*,\mathbf{b}; \text{no})] -$ (Eq 6)

$\quad [p_\lambda{}^A(\mathbf{a},*; \text{no}) \times p_\lambda{}^B(*,\mathbf{b}; \text{yes})] + [p_\lambda{}^A(\mathbf{a},*; \text{no}) \times p_\lambda{}^B(*,\mathbf{b}; \text{no})]\} - \{[p_\lambda{}^A(\mathbf{a},*; \text{yes}) \times$

$\quad p_\lambda{}^B(*,\mathbf{d}; \text{yes})] - [p_\lambda{}^A(\mathbf{a},*; \text{yes}) \times p_\lambda{}^B(*,\mathbf{d}; \text{no})] - [p_\lambda{}^A(\mathbf{a},*; \text{no}) \times p_\lambda{}^B(*,\mathbf{d}; \text{yes})] +$

$\quad [p_\lambda{}^A(\mathbf{a},*;\text{no}) \times p_\lambda{}^B(*,\mathbf{d}; \text{no})]\}$

which, can then be factored to

$E_\lambda(\mathbf{a},\mathbf{b}) - E_\lambda(\mathbf{a},\mathbf{d}) = [p_\lambda{}^A(\mathbf{a},*; \text{yes}) - p_\lambda{}^A(\mathbf{a},*; \text{no})] \times \{[p_\lambda{}^B(*,\mathbf{b}; \text{yes}) - p_\lambda{}^B(*,\mathbf{b}; \text{no})] -$ (Eq 7)

$\quad [p_\lambda{}^B(*,\mathbf{d}; \text{yes}) - p_\lambda{}^B(*,\mathbf{d}; \text{no})]\}.$

Because any given measurement must either yield a pass or fail result for polarization, the sum of the probabilities of getting either a yes or a no is equal to 1.

$\quad p_\lambda{}^A(\mathbf{a},*; \text{yes}) + p_\lambda{}^A(\mathbf{a},*; \text{no}) = 1$ (Eq 8)

This can be rewritten as

$\quad p_\lambda{}^A(\mathbf{a},*; \text{yes}) + 2p_\lambda{}^A(\mathbf{a},*; \text{no}) - p_\lambda{}^A(\mathbf{a},*; \text{no}) = 1$

$\quad p_\lambda{}^A(\mathbf{a},*; \text{yes}) - p_\lambda{}^A(\mathbf{a},*; \text{no}) = 1 - 2p_\lambda{}^A(\mathbf{a},*; \text{no}).$ (Eq 9)

The probability of $p_\lambda{}^A(\mathbf{a},*; \text{no})$ must lie between 0 and 1 as the probability of obtaining any given measurement cannot be less than 0 or greater than 1. Knowing this we can see that the right side Equation 9 must equal or lie between -1 and 1. Thus we can rewrite the absolute value of Equation 9 as an inequality

$$-1 \leq p_\lambda{}^A(\mathbf{a},*; yes) - p_\lambda{}^A(\mathbf{a},*; no) \leq 1,$$

or equivalently

$$| \, p_\lambda{}^A(\mathbf{a},*; yes) - p_\lambda{}^A(\mathbf{a},*; no) \, | \leq 1. \qquad \text{(Eq 10)}$$

If we then reconsider Equation 7 in terms of the above inequality we can remove the $[p_\lambda{}^A(\mathbf{a},*; yes) - p_\lambda{}^A(\mathbf{a},*; no)]$ term from the product and think instead of multiplying by a number that is less than or equal to one. However, we have to keep in mind that this modification will concern only the absolute values of each side of Equation 7 as we are multiplying by the absolute value of the $[p_\lambda{}^A(\mathbf{a},*; yes) - p_\lambda{}^A(\mathbf{a},*; no)]$ term. In dealing with absolute values we should keep in mind the rule $|x*y| = |x|*|y|$. Thus we obtain,

$$| \, E_\lambda(\mathbf{a},\mathbf{b}) - E_\lambda(\mathbf{a},\mathbf{d}) \, | \leq \; | \, \{[p_\lambda{}^B(*,\mathbf{b}; yes) - p_\lambda{}^B(*,\mathbf{b}; no)] - \qquad \text{(Eq 11)}$$

$$[p_\lambda{}^B(*,\mathbf{d}; yes) - p_\lambda{}^B(*,\mathbf{d}; no)]\} \, |.$$

Similarly we can think of the above Equation 11 in terms of an observable $\mathbf{c}$ in which we find the sum of the terms,

$$| \, E_\lambda(\mathbf{c},\mathbf{b}) + E_\lambda(\mathbf{c},\mathbf{d}) \, | \leq \; | \, \{[p_\lambda{}^B(*,\mathbf{b}; yes) - p_\lambda{}^B(*,\mathbf{b}; no)] + \qquad \text{(Eq 12)}$$

$$[p_\lambda{}^B(*,\mathbf{d}; yes) - p_\lambda{}^B(*,\mathbf{d}; no)]\} \, |.$$

Now if we consider the sum of Equations 11 and 12,

$$| \, E_\lambda(\mathbf{a},\mathbf{b}) - E_\lambda(\mathbf{a},\mathbf{d}) \, | + | \, E_\lambda(\mathbf{c},\mathbf{b}) + E_\lambda(\mathbf{c},\mathbf{d}) \, | \leq | \, \{[p_\lambda{}^B(*,\mathbf{b}; yes) - p_\lambda{}^B(*,\mathbf{b}; no)] - \qquad \text{(Eq 13)}$$

$$[p_\lambda{}^B(*,\mathbf{d}; yes) - p_\lambda{}^B(*,\mathbf{d}; no)]\} \, | + | \, \{[p_\lambda{}^B(*,\mathbf{b}; yes) - p_\lambda{}^B(*,\mathbf{b}; no)] +$$

$$[p_\lambda{}^B(*,\mathbf{d}; yes) - p_\lambda{}^B(*,\mathbf{d}; no)]\} \, |.$$

To simplify this we can assign variables to the following values,

$$r = p_\lambda{}^B(*,\mathbf{b}; \text{yes}) - p_\lambda{}^B(*,\mathbf{b}; \text{no}) \qquad\qquad\qquad \text{(Eq 14)}$$

$$s = p_\lambda{}^B(*,\mathbf{d}; \text{yes}) - p_\lambda{}^B(*,\mathbf{d}; \text{no}) \qquad\qquad\qquad \text{(Eq 15)}$$

and can rewrite this sum as

$$| E_\lambda(\mathbf{a},\mathbf{b}) - E_\lambda(\mathbf{a},\mathbf{d}) | + | E_\lambda(\mathbf{c},\mathbf{b}) + E_\lambda(\mathbf{c},\mathbf{d}) | \leq | r - s | + | r + s |. \qquad\qquad \text{(Eq 16)}$$

Let us consider all of the possible values that can be obtained from the $| r - s | + | r + s |$ considering that the absolute value of both r and s must be equal to or between -1 and 1 as stated in Equation 10. If we construct a table we can see what the maximum values of $| r - s | + | r + s |$ are by examining each case at which r and s are at their minimum and maximum values, -1 and 1.

r	s	$| r - s | + | r + s |$
-1	-1	$\| 0 \| + \| \text{-}2 \| = 2$
-1	1	$\| \text{-}2 \| + \| 0 \| = 2$
1	-1	$\| 2 \| + \| 0 \| = 2$
1	1	$\| 0 \| + \| 2 \| = 2$

The table above illustrates that the maximum value for which $| r - s | + | r + s |$ can equal is 2. So we can rewrite Equation 16 as,

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$$| E_\lambda(\mathbf{a},\mathbf{b}) - E_\lambda(\mathbf{a},\mathbf{d}) | + | E_\lambda(\mathbf{c},\mathbf{b}) + E_\lambda(\mathbf{c},\mathbf{d}) | \leq 2. \tag{Eq 17}$$

We now seem to have an inequality that tells us something useful about our system. Since λ describes the entire state of the system, including explicit and hidden variables, we must average the quantities being measured over all of the variables that are described by the state of the system λ. To do so, we find the average of the function of the form $E_{\lambda i}(\mathbf{m},\mathbf{n})$ for N_i particles over a total of N particles characterized by λ_i,

$$E_\lambda(\mathbf{m},\mathbf{n}) = \Sigma_i\, N_i/N\, E_{\lambda i}(\mathbf{m},\mathbf{n}). \tag{Eq 18}$$

This new value of $E_\lambda(\mathbf{m},\mathbf{n})$ will give the average value of $E_{\lambda i}(\mathbf{m},\mathbf{n})$ over the entire ensemble. We can now rewrite the left side of Equation 17 keeping such averages in mind,

$$| E_\lambda(\mathbf{a},\mathbf{b}) - E_\lambda(\mathbf{a},\mathbf{d}) | + | E_\lambda(\mathbf{c},\mathbf{b}) + E_\lambda(\mathbf{c},\mathbf{d}) | \tag{Eq 19}$$

$$= | \Sigma_i\, N_i/N\, [E_{\lambda i}(\mathbf{a},\mathbf{b}) - E_{\lambda i}(\mathbf{a},\mathbf{d})] | + | \Sigma_i\, N_i/N\, [E_{\lambda i}(\mathbf{c},\mathbf{b}) + E_{\lambda i}(\mathbf{c},\mathbf{d})]$$

$$= \Sigma_i\, N_i/N\, \{| E_{\lambda i}(\mathbf{a},\mathbf{b}) - E_{\lambda i}(\mathbf{a},\mathbf{d}) | + | E_{\lambda i}(\mathbf{c},\mathbf{b}) + E_{\lambda i}(\mathbf{c},\mathbf{d}) |\}.$$

Examining the results of these calculations we see first that $\Sigma_i\, N_i/N = 1$. This is because the sum of all particles, N_i, divided by the total number of particles must equal unity. This leaves us with the familiar result,

$$\Sigma_i\, | E_{\lambda i}(\mathbf{a},\mathbf{b}) - E_{\lambda i}(\mathbf{a},\mathbf{d}) | + | E_{\lambda i}(\mathbf{c},\mathbf{b}) + E_{\lambda i}(\mathbf{c},\mathbf{d}) | = | E_\lambda(\mathbf{a},\mathbf{b}) - E_\lambda(\mathbf{a},\mathbf{d}) | + | E_\lambda(\mathbf{c},\mathbf{b}) + E_\lambda(\mathbf{c},\mathbf{d}) | \tag{Eq 20}$$

$$| E_\lambda(\mathbf{a},\mathbf{b}) - E_\lambda(\mathbf{a},\mathbf{d}) | + | E_\lambda(\mathbf{c},\mathbf{b}) + E_\lambda(\mathbf{c},\mathbf{d}) | \leq 2$$

We have again obtained $| E_\lambda(\mathbf{a},\mathbf{b}) - E_\lambda(\mathbf{a},\mathbf{d}) | + | E_\lambda(\mathbf{c},\mathbf{b}) + E_\lambda(\mathbf{c},\mathbf{d}) | \leq 2$, and thus we obtained proof of Bell's Inequality.

Section VI.3 Testing the Inequality

If we now take the inequality and try to apply it to real measurable values we will see that the inequality is violated. Let us look at the probabilities involved in testing the polarization states of two photons. The probability of both of the photons getting the same results for any given measurement will be

$$p_\lambda^{AB}(\mathbf{a},\mathbf{b};\ \text{yes, yes}) = p_\lambda^{AB}(\mathbf{a},\mathbf{b};\ \text{no, no}) = \frac{1}{2}\cos^2(\theta) \qquad \text{(Eq 21)}$$

and the probability of obtaining dissimilar results for any given measurement will be

$$p_\lambda^{AB}(\mathbf{a},\mathbf{b};\ \text{yes, no}) = p_\lambda^{AB}(\mathbf{a},\mathbf{b};\ \text{no, yes}) = \frac{1}{2}\sin^2(\theta) \qquad \text{(Eq 22)}$$

where θ denotes the angle between $\mathbf{a}$ and $\mathbf{b}$. Equations 21 and 22 have been found by applying Malus' law which states that the intensity of a light beam that passes through a polarizer is, $I = I_0\cos^2\theta_i$. I_0 describes the initial intensity of the beam and θ_i describes the angle between the light's initial polarization direction, n, and the axis of the polarizer, n'. Malus' law tells us that that the probability for a photon polarized in a direction n to pass through an ideal polarization analyzer with axis of transmission n' equals the squared cosine of the angle between n and n'. Thus when we want to describe similar results as in equation 21, we want to use the same equation given by Malus' law. When the results are dissimilar as in equation 22, we can think of the fraction of the beam which passes through the polarizer as $1 - \cos^2(\theta) = \sin^2(\theta)$. For dissimilar results the intensity of the beam becomes $I = I_0\sin^2\theta_i$.

If we substitute these equations into Equation 2 we see,

$$E_\lambda(\mathbf{a},\mathbf{b}) = p_\lambda^{AB}(\mathbf{a},\mathbf{b};\ \text{yes, yes}) - p_\lambda^{AB}(\mathbf{a},\mathbf{b};\ \text{yes, no}) - p_\lambda^{AB}(\mathbf{a},\mathbf{b};\ \text{no, yes}) + p_\lambda^{AB}(\mathbf{a},\mathbf{b};\ \text{no, no})$$

$$= \tfrac{1}{2} \cos^2(\theta) - \tfrac{1}{2} \sin^2(\theta) - \tfrac{1}{2} \sin^2(\theta) + \tfrac{1}{2} \cos^2(\theta)$$

$$= \cos^2(\theta) - \sin^2(\theta)$$

using the trigonometric identity $\cos^2(u) - \sin^2(u) = \cos(2u)$ we can simplify this equation to

$$= \cos[2\theta]. \tag{Eq 23}$$

If we assign values to our measurements of **a**, **b**, **c**, and **d** allowing **a** = 0˚, **b** = 22.5˚, **c** = 45˚, and **d** = 67. 5˚ we can test Bell's Inequality:

$$| \, E_\lambda(\mathbf{a,b}) - E_\lambda(\mathbf{a,d}) \, | + | \, E_\lambda(\mathbf{c,b}) + E_\lambda(\mathbf{c,d}) \, | \leq 2$$

$$| \cos[2\theta_{\mathbf{ab}}] - \cos[2\theta_{\mathbf{ad}}] \, | + | \cos[2\theta_{\mathbf{cb}}] + \cos[2\theta_{\mathbf{cd}}] \, | \leq 2$$

$$| \cos[2(22.5˚)] - \cos[2(67.5˚)] \, | + | \cos[2(-22.5˚)] + \cos[2(22.5˚)] \, | \leq 2$$

$$| \, (1/\sqrt{2} + 1/\sqrt{2}) \, | + | \, (1/\sqrt{2} + 1/\sqrt{2}) \, | = 4/\sqrt{2} \nleq 2$$

As we see inserting these measurable values into Bell's Inequality give us $4/\sqrt{2}$, which is greater than 2. Thus, we have violated Bell's Inequality.

Section VI.4 Derivation Using Spin

Similarly to the experiment conceived by David Bohm we can model the Bell Inequality using spin measurements. This derivation has been modeled off a version of Bell's Inequality using spin written by Bernard D'Espagnat. It begins by resting upon the assumption that quantum mechanics is incomplete and incorporates hidden variables. This means that when measurements of an observable are taken for two particles their state function is characterized by the presence of hidden variables, in order to account for any extra observables that may be

missing from the picture. The complete state function of the system will then determine the outcome of whatever measurements are taken. This assumption leaves room for us to believe that the hidden variables are accountable for a type of local causation.

For the purpose of this derivation the observable being considered will be the particles' spin components. It should then be assumed that the particles possess some definite spin state, in any possible direction, prior to being measured. In order to take spin measurements we can imagine that we have a source emitting pairs of particles towards two detectors, positioned to the right and left of the source, as shown in Figure 2. When a particle enters a detector, a measurement of the particle's x, y, or z spin component is made and their spin value is displayed.

Source

Detector 1   Detector 2
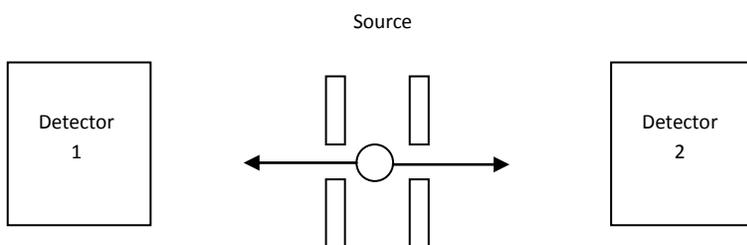
Figure 2: The figure above shows the position of the source emitting pairs of particles and the detectors set to measure the spin. This is the same setup as seen in the Bohm experiment.

The detectors cannot simultaneously measure a particle's spin on more than one axis. Such measurements are prohibited from being exactly known at a given instant according to the uncertainty principle. The uncertainty principle states that any two variables that do not commute cannot be known simultaneously. The condition for variables to commute is given by $[A, B] = AB - BA = 0$. In the case of spin variables along the x, y, and z axis we see $[S_x, S_y] = i\hbar S_z \neq 0$, $[S_y, S_z] = i\hbar S_x \neq 0$, and $[S_z, S_x] = i\hbar S_y \neq 0$. Thus, two variables of spin cannot be known

simultaneously for a particle as their commutation relations do not equal zero. So we are only able to measure spin along one of the particles' axis at a time per detector when performing such an experiment.

Experimental setups such as these have shown there to exist interesting correlations between observables of distant particles. Let us consider the case in which two particles are emitted from a single decaying particle at the source, such that the two particles have a total spin of zero. When both detectors, as in figure 2, are both set to measure the spin of a given axis, measurements always show that the left detector will obtain a measurement (+ or -) that is opposite of the right detector (- or +). For example, if the particle being measured by the detector on the left has (+) spin in the x-direction then the particle on the right will measure (-) spin in the x-direction.

With this information in mind we can now begin to derive Bell's inequality. We will let N(x+,y-) be representative of the total number of pairs with (+) spin in the x-direction and (-) spin in the y-direction during a given time interval. Because any particle can only have a (+) or (-) spin in the z-direction we can re-express N(x+,y-) with equation 24.

$$N(x+,y-) = N(x+,y-,z+) + N(x+,y-,z-) \tag{Eq 24}$$

Similarly, we can express N(x+,z-) with equation 25 and N(y-,z+) with equation 26.

$$N(x+,z-) = N(x+,y+,z-) + N(x+,y-,z-) \tag{Eq 25}$$

$$N(y-,z+) = N(x+,y-,z+) + N(x-,y-,z+) \tag{Eq 26}$$

From equation 25, we see that N(x+,z-) cannot be less than N(x+,y-,z-). This is because any particle with a N(x+,y-,z-) configuration already is included in the total N(x+,z-). We can express this in terms of an inequality as shown in equation 27.

$$N(x+,z-) \geq N(x+,y-,z-) \tag{Eq 27}$$

Similarly, we can show

$$N(y-,z+) \geq N(x+,y-,z+). \tag{Eq 28}$$

Now if we add equations 27 and 28, then substitute them into equation 24 we will see the following inequality,

$$N(x+,y-) \leq N(y-,z+) + N(x+,z-). \tag{Eq 29}$$

Because we cannot simultaneously know two components of a particle's spin we could not know the truth of the above inequality by taking measurements of one particle alone. For example, using only one detector we could not know N(x+,y-) because we could not simultaneously use it to measure x+ and y-. However, we do know that given a pair of particles emitted towards two detectors on the left and right of a source the measurement taken of the particle on the left will always yield a result that is opposite of the measurement taken by the detector on the right. This insight will allow us to discover the truth value of the inequality given in equation 29, as we can rewrite the inequality in terms of the pair of particles rather than a single particle. To do so for each configuration in equation 29 we will allow the first spin component to represent the measurement of the left detector and the second component to represent the measurement of the right detector. In allowing the second component of the configuration to account for the

measurement of the right detector we must flip the sign, as the right detector will have the opposite measurement. For example, N(x+,y-) will become N(xL+,yR+).

$$N(xL+,yR+) \leq N(yL-,zR-) + N(xR-,zL+) \tag{Eq 30}$$

Measurements of all parts of equation 30 still cannot be taken simultaneously as we cannot measure the left particle's x and y spin simultaneously nor can we measure the right particle's y and z spin simultaneously. To remedy this situation we can think of equation 30 in terms or probabilities of each configuration. To obtain such probabilities we would have to set the detectors to the desired component we wish to measure. Then we could take data on the number of times we observe the detectors as measuring this spin combination, $N_{observed}$. So we have data for each value of $N_{observed}(xL+,yR+)$, $N_{observed}(yL-,zR-)$, and $N_{observed}(xR-,zL+)$. The probability, P, for each measurement would be the number of times we observed that spin combination divided by the total number of pairs to pass through the detectors, N_{total}.

$$P = N_{observed}/ N_{total} \tag{Eq 31}$$

$$P(xL+,yR+) = N_{observed}(xL+,yR+)/ N_{total}$$

$$P(yL-,zR-) = N_{observed}(yL-,zR-)/ N_{total}$$

$$P(xL+,zR+) = N_{observed}(xR-,zL+)/ N_{total}$$

Upon finding the probability for each spin pair we can re-write equation 30 in terms of these probabilities,

$$P(xL+,yR+) \leq P(yL-,zR-) + P(xL+,zR+). \tag{Eq 32}$$

This is one form of the Bell Inequality.

This inequality has been shown to be violated when compared to empirical data. This inequality was formulated on the assumption that hidden variables may have allowed for a sort of local causality. Since this inequality has been violated we cannot assume a local interaction between the particles.

Section VI.5 Derivation Using Minimal Assumptions

In the previous derivation it was assumed that the measurement by the detector on the left side for any given direction would yield the opposite measurement of the detector on the right side. It is also assumed that hidden variables alter the particle's state function. This alteration made to the state function is what is taken to allow for local causality. However, in this derivation we will make the minimal assumption possible to account for local causality. This assumption is that the value of a given hidden variable is what determines the probability of an outcome with a determined experimental setup. The value of such a hidden variable will be represented by λ.

As in the last derivation we will consider the measurement of two particle's spin components by detectors to the right and left of a source emitting electron pairs. Instead of measuring the x, y, and z components of spin we will consider the measurement of the spin components of L and L' on the left detector and R and R' on the right detector. The outcomes again will be (+) or (-). We will let $P(+_{\text{left}}|\lambda LR)$ be representative of the probability of getting a (+) spin measurement on the left detector when the experimental setup is such that the hidden variable λ is present, the detector on the left is set to measure L, and the detector on the right is set to measure R. With this notation we can then create an equation to define the expectation value for the measurement taken by the left detector. The expectation value should be thought of

as the average value of the measurement as the measurement is taken in quick repetition. To get

this average value we must sum the product of each possible measurement and it's probability.

To then obtain the expectation value of the measurement taken by the left detector while the

detectors are set to measure L and R , $E_{left}(\lambda LR)$, we can write

$$E_{left}(\lambda LR) = (+1)*P(+_{left}|\lambda LR) + (-1)*P(-_{left}|\lambda LR)$$

$$= P(+_{left}|\lambda LR) - P(-_{left}|\lambda LR) \qquad \text{(Eq 33)}$$

To find the expectation value given by the measurement from both detectors when set to measure

L and R, we must consider the sum of the product of the measurements from each detector and

the probability of each pair of results.

$$E_{left,right}(\lambda LR) = (+1)*(+1)*P(++|\lambda LR) + (-1)*(-1)*P(--|\lambda LR) + (+1)(-1)*P(+-|\lambda LR)$$

$$+ (-1)*(+1)*P(-+|\lambda LR)$$

$$= P(++|\lambda LR) + P(--|\lambda LR) - P(+-|\lambda LR) - P(-+|\lambda LR) \qquad \text{(Eq 34)}$$

For formulating Bell's inequality it is assumed that the probability of a particle's hidden variable

having the value λ does not depend on the setting of the detectors. This means that changing the

detector's settings should not impact the source's emission of particles in certain states.

We must allow for the possibility that λ can have a continuum of values. In order to find

the probability of a given measurement we must take into account all of the possible values of λ.

The probability of obtaining a particular measurement will then be the integral over all possible λ

values of the product of the probability of having a given hidden variable, $P(\lambda)$, and the

probability of having a particular outcome with the given hidden variable λ and its experimental

setup, for example $P(+_{\text{left}}|\lambda LR)$. Thus, we can say the probability of receiving (+) spin from the left detector over all possible spin states and with the detectors set to measure L and R is

$$P(+_{\text{left}}|LR) = \int P(+_{\text{left}}|\lambda LR)*P(\lambda) \, d\,\lambda. \qquad \text{(Eq 35)}$$

Reconsidering the expectation value of the left detector in terms of all possible λ values for a given measurement we can write,

$$E_{\text{left}}(LR) = (+1)*P(+_{\text{left}}|LR) + (-1)*P(-_{\text{left}}|LR)$$

$$= P(+_{\text{left}}|LR) - P(-_{\text{left}}|LR)$$

$$= \int P(+_{\text{left}}|\lambda LR)*P(\lambda) \, d\,\lambda - \int P(-_{\text{left}}|\lambda LR)*P(\lambda) \, d\,\lambda$$

$$= \int [P(+_{\text{left}}|\lambda LR) - P(-_{\text{left}}|\lambda LR)]*P(\lambda) \, d\,\lambda$$

$$= \int E_{\text{left}}(\lambda LR)*P(\lambda) \, d\,\lambda. \qquad \text{(Eq 36)}$$

Bell makes the assumption that a hidden variable's value, λ, is the cause of any measurement's outcome. If the hidden variable causes all of the instances in which a (+) spin is obtained on the left detector and a (-) is obtained on the right detector, then the probability of obtaining the joint outcome L(+) and R(-) becomes the product of the probability of getting (+) spin on the left detector and (-) on the right detector.

$$P(+-|\lambda LR) = P(+_{\text{left}}|\lambda LR)*P(-_{\text{right}}|\lambda LR) \qquad \text{(Eq 37)}$$

Bell's final assumption allows for the probability of the measurements at the opposite sides of the detector to be independent of each other. This assumption leans on the hidden variable as the resulting cause of the measurement outcome for each particle. Thus, for equation 37 any instance

in which the hidden variable has the value λ the measurement on the left will have (+) spin. We can also say that the hidden variable having the value λ also results in the measurement of (-) in the right detector, independently of what measurement is made by the left detector. We can write

$$P(+_{left}|\lambda LR) = P(+_{left}|\lambda L) \tag{Eq 38}$$

$$P(-_{right}|\lambda LR) = P(-_{right}|\lambda R) \quad . \tag{Eq 39}$$

Substituting equations 38 and 39 into equation 37 gives us,

$$P(+-|\lambda LR) = P(+_{left}|\lambda L)*P(-_{right}|\lambda R). \tag{Eq 40}$$

If we then put the expectation value from equation 34 in terms of probabilities of a joint measurement predicted by equation 40 we find,

$$E_{left,right}(\lambda LR) = P(+_{left}|\lambda L)*P(+_{right}|\lambda R) + P(-_{left}|\lambda L)*P(-_{right}|\lambda R) - P(-_{left}|\lambda L)*P(+_{right}|\lambda R)$$

$$- P(+_{left}|\lambda L)*P(-_{right}|\lambda R). \tag{Eq 41}$$

This equation can be factored to

$$E_{left,right}(\lambda LR) = P(+_{left}|\lambda L)*[P(+_{right}|\lambda R) - P(-_{right}|\lambda R)] - P(-_{left}|\lambda L)*[P(+_{right}|\lambda R) + P(-_{right}|\lambda R)]$$

$$= [P(+_{left}|\lambda L) - P(-_{left}|\lambda L)]*[P(+_{right}|\lambda R) - P(-_{right}|\lambda R)]$$

$$= E_{left}(\lambda L)* E_{right}(\lambda R). \tag{Eq 42}$$

We can then rewrite equation 36

$$E_{left,right}(LR) = \int E_{left}(\lambda L)* E_{right}(\lambda R)*P(\lambda)\, d\lambda \quad . \tag{Eq 43}$$

Similarly,

$$E_{left,right}(LR') = \int E_{left}(\lambda L)^* \, E_{right}(\lambda R')^* P(\lambda) \, d\lambda. \qquad \text{(Eq 44)}$$

We can then take the difference of equations 43 and 44

$$E_{left,right}(LR) - E_{left,right}(LR') = \int E_{left}(\lambda L)^* \, E_{right}(\lambda R)^* P(\lambda) \, d\lambda -$$

$$\int E_{left}(\lambda L)^* \, E_{right}(\lambda R')^* P(\lambda) \, d\lambda. \qquad \text{(Eq 45)}$$

If we add the value of $\int E_{left}(\lambda L)^* \, E_{right}(\lambda R)^* \, E_{left}(\lambda L')^* \, E_{right}(\lambda R')^* \, P(\lambda) \, d\lambda$ and then subtract it, thus adding 0 to the equation, to equation 45

$$E_{left,right}(LR) - E_{left,right}(LR') = \int E_{left}(\lambda L)^* \, E_{right}(\lambda R)^* P(\lambda) \, d\lambda$$

$$- \int E_{left}(\lambda L)^* \, E_{right}(\lambda R)^* \, E_{left}(\lambda L')^* \, E_{right}(\lambda R')^* \, P(\lambda) \, d\lambda$$

$$- \int E_{left}(\lambda L)^* \, E_{right}(\lambda R')^* P(\lambda) \, d\lambda$$

$$+ \int E_{left}(\lambda L)^* \, E_{right}(\lambda R)^* \, E_{left}(\lambda L')^* \, E_{right}(\lambda R')^* \, P(\lambda) \, d\lambda. \qquad \text{(Eq 46)}$$

Simplifying,

$$E_{left,right}(LR) - E_{left,right}(LR') = \int \{[E_{left}(\lambda L)^* \, E_{right}(\lambda R)]$$

$$- \, [E_{left}(\lambda L)^* \, E_{right}(\lambda R)^* \, E_{left}(\lambda L')^* \, E_{right}(\lambda R')]\} * P(\lambda) \, d\lambda$$

$$- \int \{[E_{left}(\lambda L)^* \, E_{right}(\lambda R')] - [E_{left}(\lambda L)^* \, E_{right}(\lambda R)^* E_{left}(\lambda L')^* E_{right}(\lambda R')]\} * P(\lambda) \, d\lambda$$

$$= \int \{[E_{left}(\lambda L)^* \, E_{right}(\lambda R)] * [1 - E_{left}(\lambda L')^* E_{right}(\lambda R')]\} * P(\lambda) \, d\lambda$$

$$- \int \{[E_{left}(\lambda L)^* E_{right}(\lambda R')] * [1 - E_{left}(\lambda L')^* E_{right}(\lambda R)]\} * P(\lambda) \, d\lambda. \qquad \text{(Eq 47)}$$

Because the values we are measuring yield results of +1 or -1 the weighted average, or expectation value, of these measurements must also lay between -1 and 1. Thus we know the minimum and maximum possible values of any of the expectation values in the above equation will be -1 and 1. So we can conclude that the terms $[1 - E_{left}(\lambda L') * E_{right}(\lambda R')]$ and $[1 - E_{left}(\lambda L') * E_{right}(\lambda R')]$ will have minimum and maximum values of $[1 - 1] = 0$ and $[1 - (-1)] = 2$. We can then create an inequality out of equation 47 by maximizing the right side of the equation. By setting $E_{left}(\lambda L)*E_{right}(\lambda R)$ equal to 1 and $E_{left}(\lambda L) * E_{right}(\lambda R')$ equal to -1. The inequality becomes

$$E_{left,right}(LR') - E_{left,right}(LR') \leq \int [1 - E_{right}(\lambda R')*E_{left}(\lambda L')]*P(\lambda)\,d\lambda$$

$$- \int [1 - E_{right}(\lambda R)*E_{left}(\lambda L')]*P(\lambda)\,d\lambda. \qquad (Eq\ 48)$$

The integral is taken over all possible values of the hidden variables. But we know the probabilities of all of these possibilities must sum up to 1 as there is a 100% chance of one of the hidden variables being present. This can be shown from the general rule $\int P(\lambda)\,d\lambda = 1$. So we can write

$$E_{left,right}(LR) - E_{left,right}(LR')\ ) \leq 2 - \int [E_{right}(\lambda R')*E_{left}(\lambda L')]*P(\lambda)\,d\lambda$$

$$+ \int [\ E_{right}(\lambda R)*E_{left}(\lambda L')]*P(\lambda)\,d\lambda. \qquad (Eq\ 49)$$

Similarly to equations 20 and 21 we can write

$$E_{left,right}(L'R') = \int\ E_{right}(\lambda R')*E_{left}(\lambda L')*P(\lambda)\,d\lambda \qquad (Eq\ 50)$$

$$E_{left,right}(L'R) = \int E_{right}(\lambda R)*E_{left}(\lambda L')*P(\lambda)\,d\lambda \qquad (Eq\ 51)$$

and substituting these equations into equation 26 we find

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$$E_{\text{left,right}}(LR) - E_{\text{left,right}}(LR') \leq 2 - E_{\text{left,right}}(L'R') - E_{\text{left,right}}(L'R). \qquad \text{(Eq 52)}$$

With equation 52 we have arrived at another formulation of Bell's Inequality. Again this inequality is violated when tested experimentally. If $E_{\text{left,right}}(LR) = 1$, $E_{\text{left,right}}(LR') = -1$, $E_{\text{left,right}}(L'R') = 1$, and $E_{\text{left,right}}(L'R) = 1$, then we see $2 \nleq 0$. So we must conclude that one of the assumptions that the inequality was formulated upon is not correct. This should lead us to believe that there may be some sort non-local causality taking place on the quantum level.

VII. Factorizability

The various formulations of the Bell Inequality repeatedly fail when compared to experimental results. The one thing that all of these formulations have in common is the underlying assumption of the Bell locality requirement, also known as the factorizability condition. By taking a deeper look at what the Bell locality requirement means we can get a better understanding of what a violation of Bell's inequalities says about reality. In this section, I will describe how the factorizability condition can be divided into separate conditions: parameter independence and outcome independence.

Section VII.1 Digesting Factorizability

According to philosophers Jon Jarrett (1984) and Abner Shimony (2009), factorizability is the combination of two conditions, parameter independence and outcome independence (Cushing 1989, p. 11). Parameter independence requires that the probability of the measurement outcome at detector A is independent of the settings of detector B. Formally, if we imagine that detector A is set to measure a and detector B set to measure b the probability of the outcome x_a can be written,

$$P_{ab}(x_a) = P_a(x_a). \tag{Eq 53}$$

Similarly, we can write the probability for measurement y_b as

$$P_{ab}(y_b) = P_a(y_b). \tag{Eq 54}$$

The second condition, outcome independence, requires that the probability of the outcome at detector A is independent of the probability of the outcome at detector B. This condition can be written as

$$P_{ab}(x_a/\ y_b) = P_{ab}(x_a), \text{ where } y_b > 0 . \tag{Eq 55}$$

This condition can also be written,

$$P_{ab}(y_b/\ x_a) = P_{ab}(y_b) , \text{ where } x_a > 0. \tag{Eq 56}$$

Or the more general form of this condition is described by,

$$P_{ab}(x_a\ \&\ y_b) = P_{ab}(x_a)* P_{ab}(y_b). \tag{Eq 57}$$

Section VII.2 Non-locality Implied by Parameter and Outcome Independence

We know that the Bell inequalities are violated by experimental quantum physics. The violation of these inequalities has caused us to question whether non-locality occurs on the quantum level. To better understand what kinds of non-locality are implied from Bell's inequalities we can look to parameter and outcome independence (Cushing 1989, p. 12).

First, if we imagine that the condition for parameter independence is violated, then we would have to say that signals are being sent faster than the speed of light. Such a violation would incorporate some sort of action-at-a-distance that would not allow us to reconcile the foundations of special relativity. Action-at-a-distance can be thought of as an interaction between spatially separated objects with no mediator between the objects. An example of such action-at-a- distance is gravity. Gravity causes two spatially separated objects to be attracted to one another, which influences their motion. However, in the event that parameter independence is not violated we would not have reason to believe that signals are being sent faster than the speed of light. According to Shimony (2009), we do see that the condition for parameter independence holds in experimental quantum physics.

If we now imagine that outcome independence is violated, while parameter independence holds, we do not see the same threat to our notions of relativity. Such a violation would imply that the measurement of one particle would have to cause an instantaneous change in the spin orientation of the opposite particle. Although this may also seem to contradict our notions of special relativity, philosophers have escaped such a contradiction by proposing some sort of holism or non-separability. Such theories would allow us to believe that action-at-a-distance was not occurring, rather that there was some sort of inherent connection between the objects (Cushing 1989, p. 12).

VIII. Applications of Holism and Non-separability to Quantum Mechanics

We have now looked at the arguments for non-locality in quantum mechanics as given by Bell. We have also looked at what can be interpreted from Bell's main premise, the Bell locality requirement. It has been shown that Bell's locality requirement is a result of the combination of both outcome independence and parameter independence. The violation of either of these conditions causes a violation of the Bell Inequalities. Experimental evidence has shown that quantum mechanics does not violate parameter independence (Butterfield, Fleming, Ghirardi, Grassi 1993). Thus, we will assume that parameter independence is where the violation to Bell's locality assumption occurs. Parameter independence does not pose the same threat to our notions of locality that outcome independence does. If we accept some form of holism or non-separability we find that violation of parameter independence does not imply non-locality. Thus, by including some form of holism or non-separability in our theory of quantum mechanics we do not see any violation to Einstein locality. In this section, I will discuss various types of holism and non-separability that have been discussed in connection with quantum mechanics.

Section VIII.1 Types of Holism

In Richard Healey's paper (2009) he distinguishes between two types of holism in quantum physics: methodological holism and metaphysical holism. Methodological holism is a description of holism in terms of the best way to gain an understanding of a system's behavior. Methodological holism states that in order to understand a complex system one should look at the principles governing the whole system and not its component parts. This version of holism is helpful when considering the best way to study a complex system. Metaphysical holism is the view that the nature of a whole is not determined by the nature of its parts alone. Healey further

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divides metaphysical holism theories into ontological holism and property holism. Ontological

holism is the theory that a system is more than its components. Property holism is the theory that

the properties of a compound system are not determined by the properties of its individual parts.

Both physicists, Niels Bohr and David Bohm have been interpreted as holding some form

of ontological holism. Bohr held the belief that a property, such as momentum or position, could

not be assigned to a quantum system unless the system was set up in the presence of a complete

experimental setup designed to measure that certain property. The experiment would then have

to be carried out using this experimental setup to acquire any information about that property. In

such a setup, Bohr believed that quantum objects are not independently existing components of

the apparatus-object whole (Healey). Bohm also held these views, with the additional belief that

any collection of quantum objects by themselves would constitute a united whole. Bohm

believed that the complete specification of the state of a system, given the particles'

wavefunctions, is associated with a field that guides the particle's trajectories. Bohm used the

phrase "undivided universe" when describing his beliefs on the nature of quantum systems

(Healey).

According to Healey, property holism is the form of holism that can most clearly be

linked to quantum mechanics. Property holism describes how a physical object's properties are

not fixed by the properties of its physical parts. Property holism also examines how the relations

between these parts affect the properties among the individual parts and the properties of the

overall system. Healey describes a system's set of properties and the relations between them as a

"supervenience basis." The supervenience basis will include properties, which are the same for

all members of some domain *D*. For our purposes, we can think of domain D as a system of

interacting particles. Not all properties and relations will be described as being part of this

supervenience basis. Healey claims only "qualitative intrinsic properties and relations" of parts are included. By "qualitative intrinsic properties and relations" Healey means properties and relations that exist without regard to any other object. Healey then defines property holism as a set of physical objects from a domain *D* subject to processes *P*, whose qualitative intrinsic properties are not fully imposed onto the qualitative intrinsic physical properties and relations in the supervenience basis. More simply, Healey is asserting that the real state of a whole, defined by its qualitative intrinsic parts, is not fully determined by the real state of its parts. Paul Teller offers another form of property holism in terms of relations. He defines relational holism as the existence of non-supervening relations, which means that relations between some parts of a system do not constrain the parts' intrinsic properties. Teller's simpler formulation of relational holism came before and is entailed by Healey's property holism.

Section VIII.2 Types of Non-separability

Non-separability has been defined in multiple ways. In general it is seen as the violation of the separability principle.

> *Separability Principle:* the states of any spatio-temporally separated subsystems S_1, S_2, …, S_N of a compound system S are individually well defined and the states of the compound system are wholly and completely determined by them and their physical interactions including their spatio-temporal relations.

Some types of non-separability discussed in relation to quantum mechanics include, state non-separability, spatial non-separability, and spatio-temporal non-separability. I will discuss each of these types of non-separability in this section.

State separability discusses the interactions between systems that are in a joint state. According to state separability, the state of the system can be described by the product of each individual system. This condition was used in section III.2 when trying to separate entangled systems back into two individual states. Healey's definition of state non-separability asserts that the state of a compound physical system is supervenient over the states of its component parts. A quantum entangled state is said to possess state non-separability. We can see such non-separability in the mathematical formalism used to describe an entangled state as shown in section III.2. An entangled system is represented by a vector that cannot factorize into a product of vectors. Since the product cannot be factored, we can see that the states of the individual systems cannot be determined. If we allow Ψ_n to represent the state of an individual system than we can represent non-separability as follows,

$$\Psi_{1,2,\ldots,n} \neq \Psi_1 \otimes \Psi_2 \otimes \ldots \otimes \Psi_n .$$

As we have seen in section III, quantum entanglement provides a perfect example of state non-separability. So, state non-separability can definitely be seen as existing in quantum mechanics.

Spatial and temporal non-separability offer descriptions of the spatial and temporal relations between the components of a compound system. Healey defines spatial non-separability to be qualitative intrinsic properties of a system, again meaning properties that exist without regard to any other object, which are supervenient upon its spatially separated components and the spatial relations between its components. For our purposes we can imagine this to mean that properties of a system of interacting particles are imposed on all of the particles in the system, despite their spatial separations. Similarly, we can think of this form of non-separability in terms of both space and time. Healey defines spatiotemporal non-separability as any physical process in a space-time region R that is supervenient upon the qualitative intrinsic physical properties of

all points within that region R. Spatiotemporal non-separability is then seen to entail spatial non-separability. For our purposes, this would mean that a system's properties are imposed upon its interacting particles' properties despite space and time separation between these particles.

IX. Conclusion

     This paper has demonstrated how the inclusion of holism and non-separability into quantum theory alleviates tension between special relativity and quantum mechanics. First, the concepts of holism and non-separability were explained. Then, quantum entanglement was introduced to explain how non-separability is seen as acting in quantum systems. After the basic ideas behind these concepts were introduced, Einstein's reasoning for the incompleteness of quantum mechanics was introduced. Einstein believed that quantum mechanics couldn't be complete because it violated his notions of separability and locality. It is clear from our description of entangled particles that Einstein's separability principle is violated. Separability is not a result of the incompleteness of quantum mechanics; rather it is characteristic of quantum interactions. Einstein's locality argument is important to consider. Einstein co-published the EPR paper to argue for the incompleteness of quantum mechanics. However, his arguments were not accurately reproduced in the EPR paper. In this paper, Einstein's actual arguments for the incompleteness of quantum mechanics are discussed. This paper also examined the Bell Inequalities. This paper goes over multiple derivations of the Bell's Inequalities. When compared to the experimental findings of quantum mechanics the Bell Inequalities are always violated. This violation is said to imply that quantum systems act in a non-local manner. To better understand what can be inferred from the violation of the Bell Inequalities this paper examined the Bell locality requirement. The Bell locality can be factorized into two conditions: outcome independence and parameter independence. Parameter independence is not violated by experimental quantum physics. So, the assumption was made that the violation of outcome independence causes the violation of the Bell Inequalities. Unlike parameter independence, outcome independence does not pose a serious threat to locality. In fact, if we accept outcome

independence as the condition that violates Bell's Inequalities and also consider some sort of

holism or non-separability then, the Bell Inequalities do not imply non-locality. This paper also

considered different types of holism and non-locality that are applicable to quantum mechanics.

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